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WITH A HEART OVERFLOWING WITH
FILIAL PIETY AND GRATITUDE

THE AUTHOR

HEREBY MOST LOVINGLY

DEDICATES

THIS LITTLE WORK TO THE CHERISHED MEMORY OF
HIS REVERED

AUNTIE, BENEFACTRESS, AND FOSTER-MOTHER

MARIE ANNE STEFFLRE

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ARITHMETIC.

POINT.

1. Arithmetic, (from *αριθμός*), is the science and art of numbers.

2. A simple one thing or object is called a unit; from unus, one. To join 2 or more individual several objects, persons, or individualities means merely to bring them together in *some* manner; whilst to *unite* them will mean to resolve them into one, id est, one *something*, (from unus, one).

3. A number is either a unit alone or it is 2 or more units taken collectively,—united.

4. Different peoples and nations have, at different periods, used sundry characters with which to indicate or express numbers; often letters of their respective alphabets.

5. Notation is the art of expressing numbers by means of characters agreed upon, after some fixed, recognized system.

6. The system now most in use is often called the Arabic system of notation. It is strictly decimal throughout and employs but 12 characters, namely:

1, called (in English) one;
2, called two;
3, called three;
4, called four;
5, called five;
6, called six;
7, called seven;
8, called eight;
9, called nine;
0, called zero, (more generally naught);
∇, called (the inverted) delta;
˙, called the sign of the repetend.

Other marks and signs used in *working* arithmetic are but auxiliary and inessential, and will be duly noted in their respective places.

7. The first nine of these 12 characters are called digits, from *digitus*, a finger, (since we frequently count on our fingers).

8. The ▿ is not generally *written* (though it always *may* be), unless digits are to be placed to the *right* of it. When not written, however, its place or position is ever taken to be to the right of the right-hand written figure or character immediately. Or rather, to be exact, it always is itself the right-hand figure (expressed or understood) unless *written* in some other place.

9. The sign of the repetend, instead of being written on the line with the digits, like the 0 and the ▿ are, is written *over* the repetend, and, where it requires 2 or more figures to express the repetend, it is written over the first and also over the last of these.

10. The character in the first place to the left of the ▿ (expressed or understood) represents its own face-value in units or ones; the digit in the next place leftward represents ten times *its* face-value; the next, ten times ten times *its* face-value, and so on throughout. In this way the value of, or number represented by, any given digit is increased tenfold, or multiplied by 10 every time it is removed a place farther to the left (with reference to the ▿, of course), and decreased ten-fold, or divided by 10 every time it is moved one place farther to the right. And this always remains true of them whether they are to the right or to the left of the ▿ or are being carried *across* it. Thus 11▿11, eleven, *and* eleven one-hundredths can be read, and *is* one ten, one unit, one tenth, and one one-hundredth. And one ten, one unit, (i. e., one one), one tenth, and one one-hundredth can be, and *is* written 11▿11. Hence we call this a decimal scale or system, from *decem*, 10, that is, ten.

11. The zero or cipher, 0, represents no number at all itself, wherefore correctly called naught. It serves but as an auxiliary, (though a necessary one), by filling in all places between any digit and the ∇ where, if a digit were placed, a number differing from the one intended would forthwith be expressed. For instance, we have in this system no character to express ten. But a 1 in the *second* place leftward from the ∇ will express ten times as many as if in the first place, or ten times 1, i. e., ten, the number intended. In order to get it into said second place, however, *some* character must evidently be put into the first place, that is, between the 1 and the ∇. If, now, any *digit* be so placed, *its* value is presto added to the ten, making the expression too high. The only thing, then, that is left to be done in the premises is to fill said space with a 0.

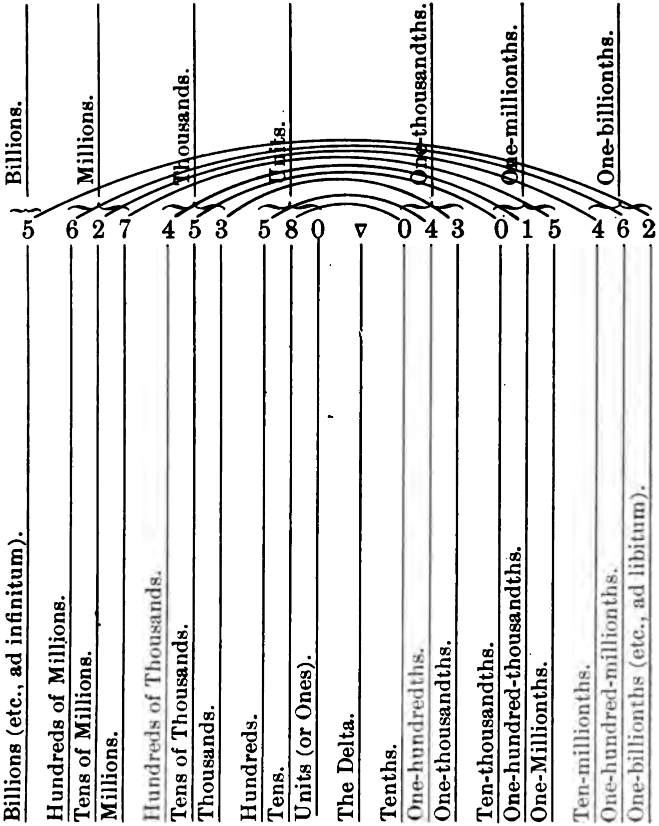
12. Numeration is the art of reading these expressions.

13. In this country we call the first place to the left of the ∇ the ones' or units' place; the next leftward, the tens' place; and the then next, the hundreds' place. Proceeding leftward, the next 3 figures are read as so many hundreds, tens, and ones of thousands, or in short, as so many thousands. In the same manner the then next 3 are read as so many millions; and then the next 3 as so many billions; then, in the same way, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, etc., ad libitum after the Latin numerals.

14. Likewise to the right of the ∇, the first place will be tenths; the next, rightward, one-hundredths, the next, one-thousandths; then ten-thousandths, one-hundred-thousandths, one-millionths, ten-millionths, one-hundred-millionths, one-billionths, etc., etc., ad sufficiendum. To facilitate the reading, especially of large numbers, it may be preferable, though, of course, it is never essential, to slightly increase the space at the left of every 3rd figure, counting leftward from the right to and including the v, and there recommencing the count, thus: 2 376 421v1 859; 7 621v010 000 426 502; 48v91 131 623. The 0 and the 9 digits ought always to be written on the line and all of even uniform altitude; the v, on the line, too, but of only half the altitude of the other characters.

15. The foregoing is perhaps better and more lucidly as well as more concisely mirrored in the following

NUMERATION TABLE.



Read: Five billion six hundred and twenty-seven million four hundred and fifty-three thousand five hundred and eighty, *and* forty-three million fifteen thousand four hundred and sixty-two one-billionths.

16. The part to the left of the ∇ is the integral part of the number, whilst that to the right is called the fractional part, though not a real fraction, arithmetically speaking, but rather a decimal. In reality, however, *all* are decimals, inasmuch as the scale is decimal. *Any* number can be read as constituting so many (units) of its lowest or right-hand order, be it to the left or to the right or even on both sides of the ∇. And in fact, this is the *usual* way of reading what 's to the *right* thereof.

17. In working arithmetical problems, we must either,

a., Increase, or,
b., Diminish

the numbers given in order to find those sought. This is accomplished in one or more of 4 ways.

a. In order to increase or augment any given number, we,

a., Add, or,
b., Multiply

2 or more given numbers together;

β. To diminish them, we,

a., Subtract the one from the other, or,
b., Divide the one into the other.

18. = is the sign of equality and is read simply "are" or "is," (equal to)—as the case may demand.

19. The signs used to indicate the above-mentioned 4 operations respectively are +, ×, −, and ÷. One of these 4 signs placed between 2 numbers means that the right-hand one of said 2 numbers is to be added to, multiplied by, subtracted from, or divided into the left-hand number, as the case may be. Division is also indicated by placing the number to be divided, (called the dividend), above a horizontal line, and the number to be divided into it, (called the divisor), below said line, thus, $\frac{3}{4}=3\div4$; $\frac{4}{2}=2$.

20. The sign of addition, +, is read "plus," or simply "and"; that of subtraction, —, "minus"; that of multiplication, ×, "times"; and that of division, ÷, "divided by."

To illustrate:

$6+2=8$, read, six and two are eight;
$6-2=4$, read, six minus two is four;
$6\times2=12$, read, six times two are twelve;
$6\div2=3$, read, six divided by two is three.

You will also hear such expressions as, "two from six leaves four"; "two into six goes three times," etc.

21. We must remember that the value of every digit is the number it represents To determine this it is essentially necessary to notice, in addition to its *face* value, its *local* value also, that is, its position with reference to the v.

22. Read the following:

27; 84; 50; 2; 4v03; 504; 31; 99; 1v1; 810; 267; 543v; 3v45; 180; 1000; 6 001 200v1; 88; 88v; v88; v 088; 8v8; 21 110 105; 66 100v01; 44v46; 720v5 406; 20 100; and 99 888 777 666 555v44 333 111 000 222.

23. Write in figures, (notate) the following:

Eighty-nine; fifty; seventeen; nine tenths; four hundred and thirteen thousand and seventy-one, *and* one tenth; seventy-six million fourteen thousand and three, *and* one billion two hundred and seven million and forty-six ten-billionths; eight thousand.

The 4 rules following are called the 4 primary rules and, as here laid down, are intended for the handling, primarily, of plain numbers whose ratio is 10.

ADDITION.

24. Addition is the process of uniting 2 or more numbers to determine their sum. Its sign is +.

25. The numbers or terms to be added are called addends; the result, their sum. (It may as well be here noted once for all, however, that, each problem being considered a question, it is quite common to call the result obtained in *any* arithmetical operation the "answer.")

26. 4+3=?
7+9=?
10+1=?
1+0=?
15+11=?
2+2=?
6+6=?
5+5=?
5+3=?
5+6=?
5+9=?
14+3+1=?
3+14+1=?
19+20+1=?
22+0+2=?

18+1+17=?
11+4+11=?
9+8+7=?
6+5+4=?
3+2+1=?
88+99=?
100+0=?
8+91+19+10=?
5+6+7+3+2=?
▿141+9▿2+1=?
56+65+3▿7=?
8+8+▿9+1+0=?
0+1+0+2+0+▿1=?
10+3+10+4=?
33+43+53=?

21+▿28+35=?
200+300+4=?
40+500+7=?
55+44+33=?
2+3+4+5+6=?
7+1+8+0+9=?
1▿11+2▿3405=?
2▿5+42+▿01=?
7+▿7+▿07+70=?
4▿5+5▿4+91=?
82+2▿8+1▿4=?
9▿712+217▿9=?
4▿1+101+6▿78=?
5+9+95+58+59=?
3+4+82=?

27. It would obviously be absurd to attempt to add mètre to weeks, for instance, or gramme to sheep. All the addends in a proposed problem must be numbers of, or reducible to, the same kind, or they cannot be added together. Even although the numbers used be merely abstract numbers, i. e., numbers absolute without reference to any objects or things by name, it must still be borne in mind that their each individual digit signifies so many ones or tens or thousands, or one-hundredths or ten-thousandths, or as the case may be, according to the position it may be occupying with reference to the v, and therefore is, to *that* extent, in a sense denominate or concrete. Wherefore tens must always be added to tens and to tens only; tenths, to tenths only, etc., etc. And so, too, of subtraction.

28. Since $2+3=3+2$, it is plain enough that, in adding lines, the result must be the same, whether added from left to right or from right to left; and so, too, as to columns, whether added upwards or downwards.

1+5+16+43+v3+9v1+476+4+2v09+44v+3=?
3+44v+2v09+4+476+9v1+v3+43+16+5+1=?
3+0+19+12+v8+1v9+100+7+87+11+88+v4+5=?

29. Add the following columns both ways:

376v753		8276450v01002		48643v9	
42	6	421508	87647	687	
900	4825	9081716	287	45612	7
219	34	88	8888	45	342
5	145	4176418	2034	12345	6
427	237	5010208	41	63908	7301

30. The learner will now be satisfied that it is more easy and convenient to add up and down, especially larger and more numerous numbers. And this is how it is generally done, too.

31. RULE.

a. Place the addends one under the other so that the v shall be in column. (This will invariably bring all the digits of each several denomination respectively into column, tens under tens, hundreds under hundreds, etc., since 2 figures must never be placed under one column nor one figure under 2 columns);

β. Draw a line beneath;

γ. Then add first the numbers indicated by the faces of the several digits in the right-hand column, placing the right-hand figure of their sum, the result, under it. Should it require more than one figure to express this result, add what's left of them other than said right-hand figure to the next column. This is called "carrying";

δ. Then proceed to the next column leftward and, having "carried" whatever, if anything, there may have been *to* carry, do as with the first column, and continue thus from column to column leftward to the end;

ε. Should the result of the last column require more than one figure to express it, place its right-hand figure under the column as before, and carry the rest leftward to an (imaginary) column having nothing else in it;

ζ. Copy the ▿ straight down, and *thereupon* erase all superfluous 0's.

32. Thus the answer may have more or even fewer places than any of the addends had. It may be advisable sometimes to let a beginner write down each several result *as obtained*, not *immediately* below the line in the place for the answer, but lower down. Then let him (or her) strike out the right-hand figure thereof and place it in its proper place beneath the column, afterwards erasing said stricken figure, and proceeding with the next column in the same manner, having first added to it what may have been left below, if anything.

33. ILLUSTRATIVE EXAMPLE.

Add 278, 562, 28, 101▿14, 6764, and ▿9732 together.

Operation and Explanation.

$$\begin{array}{r@{}l} 278 & \\ 562 & \\ 28 & \\ 101 & \triangledown 14 \\ 6764 & \\ & \triangledown 9732 \\ \hline 7734 & \triangledown 1132 \end{array}$$

$$\cancel{7}\;1\cancel{7}\;2\cancel{3}\;2\cancel{4}\;1\cancel{1}\;1\cancel{1}\;\cancel{3}\;\cancel{2}$$

Arrange first as in the margin. Begin at the right: 2 and nothing is 2. Write this sum, 2 down lower, strike out its right-hand figure, (2) and put it beneath the first column, i. e., beneath the column last added. Then proceed to the next column, first adding to it what was left below, in this instance nothing. 3 is 3. After having dealt with it as above, proceed to the then next column. 7 and 4 are 11. Place this below, strike out the right-hand figure, and write it beneath the column last added. Add the then remaining figure to the next column. 1+9+1=11. Treat again as the previous results have just been treated and, having copied the ▿ down just beneath where found, proceed: 1, 5, 6, 14, 16, 24; 2, 8, 10, 16, 23; 2, 9, 10, 15, 17; 1, 7. Now repeat the work in the same way, but adding downwards instead of upwards, and, if both times you get the same answer or sum, it is presumably correct. After the work has been otherwise quite finished, if any 0's appear in the answer that are not between some digit and the ▿, erase them.

34. It will be seen, and should here be further demonstrated and elucidated, that this amounts to each time dividing the sum of the column as obtained by the ratio, (here 10), putting down the remainder, if any, (or a 0, if naught), and carrying the quotient to the next column.

35.

Exercises.

1. Add five hundred, *and* three tenths; forty-two, *and* seven tenths; twenty-five; sixty-six thousand and seventeen; one hundred and eighty-seven, *and* sixteen one-hundredths; two million eleven thousand and six; and six hundred and seven billion one hundred and seventy-four thousand, *and* fourteen one-hundred-millionths.

2. a. 9876244v3 b. 6263045v8947

146	32	141516	17184
4013		30004	
8176066	2	6881887	0006
10101	12	148977	66
100000	01		7891
888	88	96214	
9		312	001
	42	5678	99994
16718	01	123758	12
6088	88 = ?	41000	005 = ?

3. Ole Thompson threshed in one year 471 Hektolitre of wheat, 203, of oats, 78, of flax, 21, of pease, 111, of rye, and 100, of barley. How many Hektolitre of grain did he thresh altogether that year?

4. Daniel Sweeney started from home on horseback and, traveling due east, went, on Monday 81 Chilomètre; he went next day 74 Chilomètre; on Wednesday he went 97v1 Chilomètre; on Thursday he traveled 84v4 Chilomètre; on Friday he rode 69 Chilomètre; and on Saturday he went 88v5 Chilomètre. How far from home was he then?

5. In a stage coach there were 9 passengers. One of them weighed 92 Chilogramme; the next, 83v92; the next, 100;

the next, 54v034; the next, 99v1; the next, 54v5; the next, 79; the next, 88v8; and the ninth, weighed 76 Chilogramme. The driver weighed 69v2 Chilogramme and he had on mail matter weighing 43v61 Chilogramme.

a. What did the 9 passengers weigh?

b. What did the entire load minus the driver weigh?

c. What did the entire 10 persons weigh?

d. What was the weight of the entire load driver and all?

6. New York has 12 735 030 Hektare; New Jersey, 2 024 085; Pennsylvania, 11 710 685; Delaware, 325 950; and Maryland, 3 162 390. How many Hektare in these 5 states?

7. Charles Lennan sold 100 Hektare of land for one party; 75v5, for another; 2091v45, for another; and 309v5, for a fourth party. How much land did he sell?

8. A market gardener planted 49 are to cabbage; 32v2, to tomatoes; 1v85, to strawberries; v165, to pepper; 1, to lettuce; 6v1, to carrots; 25v5, to onions; 81v403, to pease; and v5, to radishes. How much ground had he then planted?

9. From Boston to Albany is 323 Chilomètre; from Albany to Buffalo, 476; from Buffalo to Toledo, 474; from Toledo to Chicago, 389. What is the distance from Boston to Chicago over this route?

10. From Montréal to Toronto is 533 Chilomètre; from Toronto to Guelph, 77; from Guelph to Port Sarnia, 190. Required the distance on this route from Montréal to Port Sarnia.

11. What is the sum of 26, 50v3, 9, 888v808, 15v1, and 92?

12. How many are 600, 7014, 7, 2, 91, and 5640v1082?

13. Find the sum of 67, 9764, 2v35, 682, 11, 10, 960, and 51.

14. What is the sum of ninety-nine one-thousandths;

eighty-eight thousand; forty-four billion; five hundred and fifty-five ten-millionths; and six thousand and seven, *and* seventy-seven one-hundred-thousandths?

15. A hunting party shoot 201 hares, 44 ducks, 15 wolves, and 38 geese.

a. How many animals did they shoot?
b. How many of these were birds?

16. In Tralee there are 10984 men, 11208 women, 9999 girls, and 2001 boys. How many inhabitants in all?

17. A certain Frenchman is 9 years older than his wife, who is 22 years older than their oldest daughter aged 24. Required his age.

18. At an auction sale, one horse was sold for seventy-two dollars, another for twenty-one dollars and eighty cents, and another for 105 dollars. A cow went for seventeen dollars and fifty cents; and a pair of oxen for ninety-eight dollars. How much was realized in toto?

19. Going to town one day, I spent 1v16 dollars for tea, 7 dollars for shoes, 186v25 dollars for a team, 22 dollars for coal, 69 dollars for a wagon, 17 dollars for harness, and v9 dollars for sugar. How much did I spend?

20. In building his dwelling, a gentleman paid for material for the foundation, 50 dollars; for constructing same, 33 dollars and fifty-five cents; for lumber, 587 dollars; for nails, etc., 18 dollars and 25 cents; for doors and windows, 100 dollars; for carpenter work, 206 dollars and ninety cents; and for painting, seventy-seven dollars and seventy-seven cents. What did the house cost him?

21. In Puslinch there are nine flocks of sheep. In one flock there are 263 sheep; in the next, 504; in the next, 24; in the next, 1135; in the next, 818; in the next, 75; in the next, 67; in the next, 719; and in the ninth flock there are 321 sheep. How many sheep in all?

22. In a certain city there are 7 public and 4 private schools. One of the private schools has 735 pupils; another, 394; the third, 588; and in the fourth there are 233 boys and 352 girls. The public schools average 211 pupils each.

a. How many pupils in the private schools?
b. How many pupils in the public schools?
c. How many pupils in the whole 11 schools?

23. Add 111; 9170; 5▿4; two hundred and three million and seventy-four, *and* two hundred and nineteen one-thousandths; ▿00 000 015; 1680▿15; fifty-nine thousand; and one, *and* sixty-three ten-thousandths.

24. John Nestor sowed 723 are of land to wheat; 925, to oats; 100, to rye; and 344, to barley. How much ground did he sow?

25. Add 2780▿405; ninety-two nonillion and seven million; one octillion, *and* three one-billionths; twenty-five octillion one hundred and eight septillion fifty-four sextillion one quadrillion five hundred and fifty-five trillion one hundred and one billion sixty thousand three hundred and twenty, *and* eleven one-hundredths; 7 643 220; 36 888 800 101 443; a quintillion; 734▿1 982; and 537689 1057643250066336017325879 52▿09.

26. A man paid 179 dollars for a horse, three hundred and eleven dollars and sixty cents for a flock of sheep, fifty-one dollars for a wagon, and fifty-two cents for a jack-knife. How much did he spend?

27. Add one hundred and twenty-three million four hundred and fifty-six thousand seven hundred and eighty-nine, *and* nine hundred and eighty-seven million six hundred and fifty-four thousand three hundred and twenty-one ten-billionths; 8888▿88; 4 073 642 158▿8; ▿00 019; 19; 919; and fifty-one one-trillionths.

28. A merchant paid 75▿15 dollars for hardware, three hundred and eighty-four dollars and eighty cents for rope, 51 dollars for oil, and 678 dollars for paint. How much did he pay?

29. 667∇3+4∇25+61∇082+364∇123+904∇8=?

30. Into a basket weighing 2 Chilogramme were put 1∇5 Chilogramme of coffee; ∇55, of tea; ∇71, of pepper; 10, of sugar; ∇002, of nutmegs; and 1, of prunes. What did the basket with its contents then weigh?

SUBTRACTION.

36. Subtraction is the process of finding the difference, if such there be, between two given numbers by taking the smaller from (out of) the greater. If there be no difference, subtraction will demonstrate this.

37. The number to be taken away is called the subtrahend, the number from which the subtrahend is to be taken is called the minuend, and what 's then left is styled the remainder.

38. The sign of subtraction is —, and is placed after the minuend and before the subtrahend, thus, $7-3=4$; $9-1=8$; $14-12=2$; $101-99=2$; $6-6=0$. But here again it will generally be found to be preferable to place the subtrahend *under* the minuend, rather than after it, and proceed as follows:

39. RULE.

α. Having first written the minuend, place the subtrahend under it, delta under delta, and draw a line beneath;

β. Beginning now at the right, deduct the number expressed by the figure in the subtrahend from the number expressed by the figure in the minuend immediately over it, writing the remainder beneath it. When there is no (other) remainder, put down a 0. Should there be but one figure in a column, supply an (imaginary) 0;

γ. Proceed then to the next column leftward and repeat the same operation, and so on leftward to the end, taking care to copy the ▿ down into the answer or remainder just beneath where found above;

δ. Should the number expressed by a figure in the subtrahend exceed that expressed or indicated by the figure in the minuend immediately above it, borrow 1, fancy it written before the latter, and then subtract;

ϵ. Whenever you have thus borrowed 1, be sure, before proceeding further with the next column, to first add 1 to the figure therein in the subtrahend. Carry only and immediately after having borrowed;

ζ. If after the work is otherwise completed, any 0's should appear to be not between some digit and the v, they should now be erased.

> NOTE. — *Whenever it will have become necessary to borrow* 1 *as above, the then remainder will be equal to the excess of* 10 *over the difference between the face value of the digit in the subtrahend, (increased by the* 1 *to carry if such there had been), and that of the figure in the minuend.*

40. *Explanation.*

The ratio being uniformly 10, the 1 you "borrow" from any column to take it to the right must needs become 10 there. But 10 added to any single one figure is equivalent to that figure having a 1 written before it, as see: $10+8=18$; $6+10=16$; $10+0=10$, etc., etc. Again, having borrowed 1 from the left-hand nearest figure in the minuend and thus reduced it by 1, the remainder in that column must now be 1 less than it otherwise would have been. Now, inasmuch as $6-3=7-4$, for instance, and vice versa, the same result will be obtained if, instead of conceiving the minuend to be 1 less, we leave it as it is, and add 1 to the subtrahend instead, to which mode the author would give the preference. But never bother a pupil that has already acquired one good way of doing a thing, with another way. Not even where such other way may truly be a better way, unless the advantage would be quite considerable.

Of course, when handling numbers having a ratio or ratios other than 10, as, for instance, numbers expressive of a succession of varying units of time, adding the 1 borrowed will not always be equivalent to placing a 1 before the figure to which it is borrowed. In addition, too, it will be noticed that placing the right-hand figure of the sum of any column beneath and carrying the balance is, as already indicated, equivalent to dividing said sum by 10, (the ratio), placing the remainder under the column, and carrying the quotient. And this is what is done whatever the ratio. When, however, said ratio is *not* 10, it will not do to merely so put down the right-hand figure and carry the other or others, inasmuch as the division is then most apt to give us a different quotient and a different remainder. So, too, with the 1 borrowed in subtraction. Ditto when carrying in multiplication and division.

In short, the number carried to the right or borrowed must be multiplied by the ratio in the premises; the number to be carried to the left will ever be the quotient obtained by dividing the sum or product obtained by the ratio.

41. EXERCISES.

1. A railroad train starting from Crieff travels 811 Chilomètre in one day. Andrew Gerup starts afoot from the same point at the same time, going in the same direction, and finds himself at the end of the day 35 Chilomètre from Crieff. How far ahead of him is the train now?

2. The Amazon River is 640 Myriamètre long; the Mississippi, 496. Find the difference.

3. A dealer bought 2114 stère of wood and then sold to Julia Wheeler 25, to Mabel Hall one hundred, to Jennie Rafferty forty, to Agnes Traverse three hundred, *and* seventy-five one-hundredths stère. How much wood had he then left?

4. One of my horses weighs 458497 gramme, the other, 587014. Find the difference.

5. Canada contains 9 783 800 Myriare, the "United States," 9 210 800. Required the difference.

6. The Empire of Russia comprises 22 429 500 Myriare. How much is it larger

a. Than Canada?
b. Than the United States?
c. Than Canada and the United States combined?

7. Lake Baikal covers three hundred and twenty-three million seven hundred and fifty thousand three hundred are; Lake Titicaca, 103 600 000. Find the difference.

8. The Pacific Ocean covers 181 301 000 Myriare; the Arctic Ocean, 10 360 100. What is the difference?

9. Mount Ararat is five thousand three hundred and sixty-one mètre high; Mount Sinai, two thousand six hundred and fourteen, *and* one-tenth mètre. Find the difference.

10. Alice Grenier buys 2015 Chilogramme of salt, and then sells 1928 Chilogramme of it to Mary McLean. How much salt has she still left?

11. From 6 take 1.

12. 57—23=?

13. 14—9=?

14. 203—18=?

15. 33—21=?

16. 1116542
321043
?

17. 364815670
230541862
?

18. 14098154
81323
?

19. 41413450—9=?

20. 3415163
185752
?

21. 167544123
92076412
?

22. 9594132
1314153
?

23. 32675▿4
6779▿1
?

24. 76▿5—42=?

25. 8654
2142
?

26. 6564▿3—828▿8=?

27. 95949▿8
67421▿1
?

28. 766554▿2
33288▿7
?

29. 8876462
7854685
?

30. 25▿87
5▿264
?

31. 88▿645
21▿415
?

32. 148▿2
60▿09
?

33. 1450v1
888v97
?

34. 87v6001
44v89
?

35. 53648v21
18514v34
?

36. 4142434v7639
2761058v101
?

37. 870000000v004
78000010v06
?

38. 33338865072
21202900857
?

39. 12131v415161718
1910v10111213
?

40. 1718191051v12304
567890012v34567
?

41. 567v
9v87868584838
?

42. 5v67895432343
v98991045
?

43. 7v37656725
1v57625419
?

44. 91270304
2123242v5
?

45. 2876554433v21
918760000v07
?

46. 7876775
7170707
?

47. From six billion eighty-one million seven hundred and five thousand four hundred and forty-three, *and* eighteen thousand and fourteen one-hundred-millionths take nine hundred and seventy-six million four hundred and one thousand and one, *and* sixty-five one-hundredths.

48. Beginning with the lowest, from every odd number between 10 and 30 take 11 successively.

49. From 1 take successively every digit, beginning with the highest, placed in the tenths' column.

50. From 2, beginning with the lowest, take every even digit placed in the one-hundredths' column.

51. How many is 47623▿81 more than 888▿17?

52. What is the difference between 76▿888 888 882 and 48▿671?

53. What number added to 99675440▿1 = 953116915?

54. 362 634 584 900 105--487▿64 325 154 327 = ?

55. Robert Crame bought a farm for $2684▿4 and sold it afterwards for $2906▿03. How much did he thus gain?

Note.—"$" *is called the dollar-mark, and means dollars.*

56. The earnings of a certain party in one year amounted to $1000, whilst his gain in said time was $4976▿12. How much more did he gain than he earned in that year?

57. A certain gambler "made," that is, increased his wealth, to the tune of $3000000 in one year, *earning* in the same period $0. How much more did he make than he earned?

58. Louis Ellena earned in 5 years $2391 and received therefor $902▿15. Required his loss.

59. Tom Ward bought dry goods for $5173 and groceries for $4007▿17. He sold the dry goods for $5113▿41 and the groceries for $7164▿94. Did he gain or lose? and how much?

60. A certain candidate for office *expected* to get 2001 votes. He *got* 999. To what extent were his expectations an exaggeration?

61. A certain teacher earned in one year one thousand and fifty dollars. He was paid $188 in money and the balance in abuse. How much was he paid in abuse?

62. Subtract ▿1 from 91.

63. Subtract 3▿1 from 4▿88.

64. Subtract 27 from 92▿5.

65. A certain pettifogger received from his victim $500 to advocate his interests in a certain proceeding, then sold out to the opposite party for $600, damaging his said victim $3510. How much more did he *get* than he really *earned?*

66. John Rhodes bought 278514v5 Hektogramme of sugar. How much of it did he have left after selling one hundred and nine thousand four hundred and eleven, *and* three hundred and thirty-five one-thousandths Hektogramme?

67. A gentleman died in 1867, aged 83 years. In what year was he born?

68. A certain laborer received for his year's work $321, whilst his family expenses were $514v88. How much did he run behind?

69. Harry Gidley paid $2111 for a farm, $710v9 for implements, and four hundred and forty-four dollars and twenty-five cents for stock. He also built upon said farm a house which cost him $800. Thereupon he sold out for four thousand three hundred and seventy-five dollars. Did he gain or lose? and how much?

70. John McLean sold a team for $327v19, realizing thereon a profit of $100. What had the team cost him?

71. The Emperor of China has four hundred million subjects, the Queen of England, three hundred and twenty million. In the United States there are, 80 000 000. Required the excess of the population of the Chinese Empire

a. Over the British Empire's;
b. Over the United States'.

72. Georgia has one million eight hundred and thirty-seven thousand three hundred and fifty-three inhabitants, of whom 858 996 are colored. How many white people in Georgia?

73. Europe contains 10 240 300 Myriare; Australia, 8 770-900; and South America, 17 838 400.

a. How much is Europe larger than Australia?

b. How much is South America larger than Europe?

c. How much is South America larger than Australia?

d. How much is South America smaller than said other 2 grand divisions combined?

e. How much is Europe smaller than said other 2 grand divisions combined?

f. How much is Australia smaller than the said other 2 grand divisions combined?

g. How much land in these 3 grand divisions?

h. How much land in Europe and South America taken together?

74. A bankrupt's liabilities are seventy-five thousand and ten dollars. His assets, $19375, of which the receiver manages to consume $17200▿48. What amount of his debts remain unpaid?

MULTIPLICATION.

42. Count 30 by threes.

In doing this you take 3 ten times, and the result is 30. Thus you multiply 3 by 10.

43. The number taken or to be taken a set number of times is called the multiplicand. The number indicating how often the multiplicand is to be taken is called the multiplier, and the result obtained, the product. Thus, in the above, 3 is the multiplicand, 10, the multiplier, and 30, the product. The operation would be shown in the following expression: $10\times3=30$. Read, 3 multiplied by 10 is equal to 30. But 10 multiplied by 3 is likewise equal to 30. Hence $10\times3=3\times10$, and 3×10 can be read either as three times ten or as ten times three. And so of *any* two numbers on a line having × between them.

44. All multiplication problems could be worked out by addition, thus:

3	
3	
3	8
3	
3	8
3	
3	8
3	
24	24

It will now be seen that $x\times y=y\times x$, a fact which should be amply and clearly impressed upon the minds of all learners, and profusely, ay, even extravagantly illustrated and demonstrated before going further. $2\times3=3\times2$; that is, twice 3 is equal to thrice 2. And $4\times5=5\times4$, five times four is equal to four times five and four times five is equal to five times four, and so on in this manner invariably.

45. Multiplication is a process devised for more easily and readily adding 2 or more *equal* numbers by the use of a knowledge of the product of all possible combinations between *any* 2 of the several figures. The following table contains *all* of these products and, (remembering that $x\times y=y\times x$), all these combinations. It would be too extremely inconvenient altogether to find the sought-for result by addition when a large number is to be taken a great number of times; 2765×1932, or 8401×734, or 76 439 005v376 298×56 150v3, for instance.

46. Multiplication Table.

$x\times y=y\times x$	$x\times 0=0$	$x\times 1=x$	$x\times 10=x0$
$2\times 2=4$	$3\times 3=9$	$4\times 4=16$	$5\times 5=25$
“ “ $3=6$	“ “ $4=12$	“ “ $5=20$	“ “ $6=30$
“ “ $4=8$	“ “ $5=15$	“ “ $6=24$	“ “ $7=35$
“ “ $5=10$	“ “ $6=18$	“ “ $7=28$	“ “ $8=40$
“ “ $6=12$	“ “ $7=21$	“ “ $8=32$	“ “ $9=45$
“ “ $7=14$	“ “ $8=24$	“ “ $9=36$	$6\times 6=36$
“ “ $8=16$	“ “ $9=27$	$7\times 7=49$	“ “ $7=42$
“ “ $9=18$	$8\times 8=64$	“ “ $8=56$	“ “ $8=48$
$9\times 9=81$	“ “ $9=72$	“ “ $9=63$	“ “ $9=54$

Note.— *This table could be learned in* 4 *lessons of a column each, and then gone over again in* 11 *lessons from* 0×0 *to* 10×10, *supplemented by frequent and continued oral cross-questioning drills. Again it could be recited thus: Twice naught is naught, no times two is naught; twice one are two, once two is two; twice three are six, thrice two are six; and so on.*

JENNET.

2×2=4—
Jennet dainty gaiters wore.
2×3=6—
And made us laugh with merry tricks.
2×4=8—
She came to school without a slate.
2×5=10—
Nor did she bring a writing-pen.
2×6=12—
She fell one day and hurt herself.
2×7=14—
By-and-by there came a-courting
2×8=16—
A beardless lad from Aberdeen;
2×9=18—
But Jennie kept him waiting,
3×3=9—
Till he commenced to whine,
3×4=12—
Then told him go and drown himself.
3×5=15—
One day the snow was drifting,
3×6=18—
And we for Jane were waiting,
3×7=21—
When in she bolted on the run;
3×8=24—
Nor did she stop to close the door,
3×9=27—
But popped her feet into the oven.
4×4=16—
She was handy in the kitchen,
4×5=20—
With her dimpled hand so dainty.
4×6=24—
Of learning she had little store;

4×7=28—
But patiently she bore her fate;
4×8=32—
And glad was she for work to do;
4×9=36—
Although the fairest of her sex.
5×5=25—
You see, she knew she 'd never thrive,
5×6=30—
If indolent or dirty.
5×7=35—
She was the jolliest girl alive;
5×8=40—
And with her laugh so hearty,
5×9=45—
Our drooping spirits she 'd revive,
6×6=36—
As soon as in our crowd she 'd mix.
6×7=42—
Her hat was neat, her dress was blue.
6×8=48—
I met her at the garden gate
6×9=54—
One ev'ning as I came to woo her.
7×7=49—
I asked her if she 'd please be mine
7×8=56—
Then smacked her on her freckled cheeks.
7×9=63—
She blushed and said, my wife she 'd be
8×8=64—
If I 'd be sure to drink no more.
8×9=72—
She 's worth the price, and far more too,
9×9=81—
So henceforth I will whiskey shun.

47. Money is a fixed measure of value.

The ratio between the various terms or denominations of money used in this country is uniformly 10, according to the following

TABLE.

10 mills, (mi.) = 1 cent, (c.);
10 cents = 1 dime, (di.);
10 dimes = 1 dollar, ($);
10 dollars = 1 eagle, (E.).

Taking the dollar for the *primary* unit, our money would certainly be much better and more conveniently termed and rendered in conformity with the nomenclature of the following table on the same uniform scale or ratio.

This table has the mètre for its foundation, and is being now used in nearly all countries.

48. UNIVERSAL TABLE OF WEIGHTS AND MEASURES.

milli, (m)
centi, (c)
deci, (d)

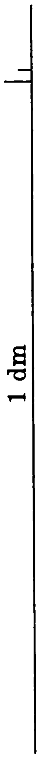
Deka, (D)
Hekto, (H)
Chilo, (C)
Myria, (M)

gramme, (g)
mètre, (m)
litre, (l)
stère, (s)
are (a)

(Scale 10.)

10 Mg = 1 Quintal, (Q); 10 Q = 1 Tonneau, (T).

NOTE.—*The final vowel of these prefixes is dropped before are.*



The mètre, from which all these primary units are deduced, is so nearly the v0 000 001 part of the distance on the face of the earth from the equator to the pole, that it may be denominated as such. The French government very carefully preserves a model, now the prototype mètre of the world. Its exact length has been traced with the utmost possible care and accuracy on a bar of 95% platinum and 5% iridium, constituting a body less susceptible to heat and cold, that is, less affected thereby as to its bulk than any other now known material substance. A model weight is also so preserved there. Even the number of light-waves in a mètre have been counted. Copies of these models, taken at a given temperature, are to be found at the (political) capital of every other country.

The gramme is the weight of a cubic centimètre of purest (distilled) water at its greatest density weighed in vacuum at sea-level in the latitude of Paris, France.

The litre is the volume of a cubic decimètre.

The stère is simply a Chilolitre, (i. e., a cubic mètre).

The are is a square Dekamètre, (or its equivalent).

49. Circular or Angular Measure.

100 second, (") = 1 minute, (');
100 minute = 1 grade, (^{g});
100 grade = 1 right-angle, (∟);
4 right-angle = 1 circle, (c.).

Note.—A " *on a meridian would therefore be Dm, practically speaking.*

50. TIME TABLE.

60 seconds, (s.) = 1 minute, (m.);
60 minutes = 1 hour, (h.);
24 hours = 1 day, (d.);
7 days = 1 week, (wk.);
365 days = 1 (common) year, (y.);
366 days = 1 leap year, (lp. y.);
4 weeks = 1 lunar month, (l. mo.);
13 lunar months = 1 year;
$\frac{1}{12}$ of a year = 1 (calendar) month, (mo.);

30 days hath September,
April, June, and November.
The rest have each got 31;
February, 28 alone;
But in leap years 29.
10 years = 1 decade, (dec.);
10 decades = 1 century, (C.).

But in computing interest, 30 days are often, yes, *generally*, taken as one (calendar) month. And unless otherwise specified, a "month" means a calendar month, and a "year" a common year.

Every year exactly divisible by 4 is a leap year, unless it be the last year of the century, (Christian era), in which case it is a leap year only if it is exactly divisible by 400. Still, the year exactly divisible by 4000 is not a leap year.

All these tables should be thoroughly mastered; and, in performing any operation in arithmetic, be it addition, subtraction, multiplication, or division, the ratios respectively appertaining to the numbers being handled must always be carefully kept in view.

51. The multiplier and multiplicand are factors, (makers) of the product. Any 2 or more numbers constitute factors of the number obtained by multiplying them together. If they are prime numbers, they will be prime factors. But 1 ought not to be called a "*factor.*"

52. RULE FOR MULTIPLICATION.

α. Reserve the one of the 2 given terms for multiplier which you see will make the more convenient multiplier and place it under the other so, that the right-hand digit will be under the right-hand digit;

β. Should either term have any 0's to the right of its right-hand digit and not being in a repetend, ignore them until the work is otherwise finished and then annex them to the answer (on the right); or, better yet perhaps, count them (all in both), then write the number expressing the total number of these 0's in a corner (of the slate). Then erase said 0's and proceed without them. Having finished the work of multiplying without them, affix as many 0's to the product as were so erased as directed in ι below;

γ. Multiply the right-hand digit (or figure) of the multiplicand first by the right-hand digit of the multiplier, (a line having been drawn beneath them), and put the right-hand figure of the product thus now obtained directly under the digit of the multiplier which is being used;

δ. Then proceed leftward and multiply the next figure of the multiplicand by the same digit and add to the product you now get all of the last preceding product bereft of its right-hand figure;

ε. Place, now, the right-hand figure of the product or sum thus obtained in the next place in the product or answer to the left of the last preceding figure placed therein;

ζ. Repeat this operation step by step leftward to the end;

η. Then recommence, using the next digit to the left in the multiplier and again placing the first figure obtained by *it* under *it;*

θ. Use all the digits in the multiplier in this way in succession;

ι. Then, having drawn a line beneath the products thus obtained, add them together as they now stand and annex the number of 0's that had been erased, if any, to the right of the sum to complete the product;

κ. Count the number of "decimal places," i. e., the number of places to the right of the ∇, in the multiplicand, add thereto the number of those in the multiplier, and then count off from the right of the completed product a number of places or figures equal to the sum of these 2 numbers, placing the ∇ before said places or figures, i. e., to the left of them;

λ. Having thus located the ∇, erase, for appearance' sake, any 0's that may now be found elsewhere than between it and some digit unless they be in a repetend;

μ. If either given term be simply a 0, *it* will be the product, if a 1, the other given term will be the product if said 1 be in the units' or ones' place, but if it is to the right of the ∇ having nothing or nothing but 0's between it and the ∇ and no other digits in the term, move the ∇ in the other term as many places leftward as there are places or figures (other than the ∇) in the first named term, and if it be any power of 10 move the delta as many places to the right in the other term as said power has 0's,—affixing always or erasing or prefixing 0's when required.

ν. In case either given term is composite the other term may be multiplied by one of its factors, the product thus obtained by another, and so on, using each factor once.

53. ILLUSTRATIVE EXAMPLES.

6704588970375585v0737×0=0.
547621894×1=547621894; 6v0043×0=0.
82764v51328402×1=82764v51328402.

4v1×376=?

Operation.

376
4v1
376
1504
1541v6

6v25×474131=?

Operation.

474131
6v25
2370655
948262
2844786
2963318v75

150v52×v817=?

Operation.

150v52
v817
105364
15052
120416
122v97484

25v7×140=?

Operation.

25v7 (1
14
1028
257
3598v$\cancel{0}$

Explanation.

140 makes a more convenient multiplier than 25v7, so we take it for multiplier. Erase the 0, it being on the right, and place a "1" in the corner to show how many 0's have been erased. Now commence at the right. Four times seven are twenty-eight. Place the 8 under the 4 and reserve the 2 to be added to the next product you get. 4×5=20. 20+2=22. Repeat this operation to the end. Then do the same with the 1, writing the first figure *it* will produce, (7), under it. Thereupon we add these 2 products together as they stand, place a 0 to the right of their sum, and then point off one decimal place, there being a "1" in the bracket above. This brings the delta in the product between the 8 and the 0. Finally we erase this 0.

54. 268×4=?

Operation.

$$\begin{array}{r} 268 \\ 4 \\ \hline 1072 \end{array}$$

$$\begin{array}{l} 3\cancel{2} \\ 24 \\ \hline 2\cancel{7} \\ 8 \\ \hline 1\cancel{0} \\ 0 \\ \hline \cancel{1} \end{array}$$

670526×3=?

Operation.

$$\begin{array}{r} 670526 \\ 3 \\ \hline 2011578 \end{array}$$

$$\begin{array}{lll} 2\cancel{1} & 1\cancel{5} & 1\cancel{8} \\ 18 & 0 & 6 \\ \hline 2\cancel{0} & \cancel{1} & \cancel{7} \\ 0 & & \\ \hline \cancel{2} & & \end{array}$$

5800000×10=?

Operation.

$$\begin{array}{ll} 58 & (6 \\ 1 & \\ \hline 58000000 & \end{array}$$

$$\begin{array}{ll} \cancel{5} & \cancel{8} \end{array}$$

843×100=?

Operation.

$$\begin{array}{r} 843 \\ 100 \\ \hline 84300 \end{array}$$

Or simply 843 with two 0's affixed.

672v576×1000=?

Operation.

$$\begin{array}{r} 672v576 \\ 1000 \\ \hline 672576v000 \end{array}$$

Or simply 672576v.

v004×100=?

Operation.

$$\begin{array}{r} v004 \\ 100 \\ \hline 00v400 \end{array}$$

Or simply v4.

55. 2×3×5=6×5,=2×15,=10×3,=2(3×5),=(2×3)5.
23×42= 966;
23×2×3×7= 966;
6×7×23= 966.

Again, 14×30=2×7×3×5×2. 41×v001=v041.
18×21=2×3×3×3×7. Or 3×7×18=2×9×21, etc.

NOTE.—*A power of a number is the number or product obtained by using the given number one or more times*, (with *nothing* else), *as factor.*

Thus 6 is the first power of 6; 36, (i. e., 6×6), is its 2d power; 216, (i. e., 6×6×6), is its third power; 1296, (i. e., 6×6×6×6), is its fourth power, and so on.

These powers are indicated by small figures written to the right of the number and higher up, as will more fully be shown later. $6^3=6\times6\times6$; $6^1=6$; $6^4=6\times6\times6\times6$; $7^2=7\times7$.

56. Exercises.

1. On 99 Ha a farmer raised 11 Hl of wheat per Ha. How much wheat?

2. A man killed 15 hogs that averaged in weight 164 Cg apiece. What was their combined weight?

3. How many m. in a wk.?

4. How many ca will there be in a rectangular piece of ground 210 m long by 15v6 m wide? (Length×width.)

5. When a plane rectangular quadrilateral figure is just as wide as it is long, it is called a square. How far will it be round a square field of which one of the sides is 721 m?

6. How many ″ in 2 ∟?

7. How many cl in 27v625 s?

8. How many cg in 2 T?

9. How many mm in 4 Mm?

10. How many cl in 5v1 Cl?

11. At $20 apiece, what will 17 steers cost?

12. At $25 apiece, what will 15 calves cost?

13. At $31v15 apiece, what will 6 cows cost?

14. If 100 members of a parish give each $100 towards the erection of a church, how much of a building fund will that make?

15. Multiply 376 by 210.
16. Multiply 688v4 by 9.
17. Multiply 421 by 100.
18. 1v762×302=?
19. 6203000×4v2=?
20. 671000×2=?
21. Multiply 100000×40.
22. 61v012131×v0042=?
23. v00005×36=?
24. 8817625v4407×5v4=?
25. 839 multiplied by 93=?
26. v93 multiplied by 39=?
27. Multiply 167 by 72.
28. Multiply v39 by v93.
29. 678×4v37625=?
30. 3721v05×v01=?
31. 1v002×101=?
32. 676689606v12×1=?
33. 45v126776×0=?
34. 242v76504×v7=?
35. Multiply v1 by v01.
36. v589×v1764=?
37. 246v75×3v71=?
38. 25v5407×1v1=?
39. 666646×7v6=?
40. 100v001×2=?
41. 8008v08×56000=?
42. 6500067×761735=?

43. Multiply two million six hundred and forty-four thousand and thirty-seven, *and* eighty-seven thousand one hundred and five one-hundred-thousandths by three hundred and fifty-seven thousand, *and* sixteen one-hundredths.

44. A market gardener set out cabbage in 367 rows, putting 367 plants in each row. How many plants did he set out?

45. Next autumn our friend finds that he has just 300 marketable heads in each row, and that their average weight is 25v5 Hg each. He sells them for $v003 per Hg.

a. How much did he realize?
b. How many marketable heads were there?
c. What was their combined weight?

46. 264v09×901v2×405060×708×501=?
47. v00020304×7771v7×76×88×8v871=?
48. 8800203000×4050×56v65×81012=?
49. v0009×40000v001003004×v908807=?
50. 64v0001×v00002×765v14×915v15=?

51. 151515×51v15×11v5×820270160v1403=?
52. 123405×9×8×0×7×6×5×4×1×3×2×1v1=?
53. 256×12×987654×32v1×12345v6×7v89=?
54. 542v65289×9×0×1400×150000=?
55. 3268×5×8×10×10×786v42×10×10×10=?
56. 7864v21×111×11×8×9v1×1v02×20v1×2212v7=?

57. How many h. in a wk.?
58. What will be the product of the 9 digits when multiplied together?
59. In a certain apiary there are 15 bee-hives. In the first hive there are 1267 bees; in the next, 14000; in the next, 9874; in the next, 6101; in the next, 4170; in the next, 9111; in the next, 5111; in the next, 11162; in the next, 3100; in the next, 5555; in the next, 18118; in the next, 12317; in the next, 6066; in the next, sixty thousand and eighty-four; and in the last there are 33455 bees. How many bees altogether in the apiary?

60. How many d. in 52 wk.?
61. How many m. in 3 wk.?
62. How h. in December?
63. How many h. in February, A. D. 1900?
64. How many m. in June?
65. How many ml in 3 s?
66. How many dg in 173v2 Q?
67. How many dm in 2 Dm?
68. How many cm in a Mm?
69. How many cm in 6v251 m?
70. How many m in 25v4 Mm?
71. How many ca in 2v56 Ma?
72. How many ma in 1v01 Ha?

DIVISION.

57. Separating a number into equal groups, having first determined either the size or number of these groups, is called dividing it.

If their number has been predetermined, division will give their size, and vice versa.

The number to be divided is called the dividend; the number to be divided into the dividend is called the divisor; and the result obtained, the quotient. If there be no remainder left and nothing in the quotient to the right of the v, the division is said to be exact.

Division is indicated either by writing the dividend above a horizontal line with the divisor beneath it, thus $\frac{234}{7614}$, or by writing ÷ after the dividend and before the divisor, thus 234÷7614. If the product obtained in any multiplication operation is taken for a dividend and either of the other 2 terms for a divisor, the quotient will invariably be the remaining term. For, whenever one of the 2 factors going to make a number, (product), is divided into that number, the quotient must needs be the other factor. And multiplier and multiplicand are simply the factors of their product.

58. 4×2=?; 8÷4=?; 8÷2=?; 15÷5=?; 15×10=?; 5×10÷?; 150÷50=?; 20÷10=?; 20÷5=?; 10÷5=?; 4÷2=?; 20÷10=?; 20÷2=?; 10÷2=?; 10÷5=?; 6÷3=?; 6×3=?; 3×3=?; 18÷9=?; 8÷2=?; 8×5=?; 2×5=?; 40÷10=? So if both the divisor and the dividend be multiplied or divided by the same number, it will not alter or affect the quotient.

59. 33÷11=?; 33×2=?; 33×3=?; 66÷11=?; 99÷11=?; 14÷7=?; 14×5=?; 14×10=?; 70÷7=?; 140÷7=?

So that multiplying (or dividing) the dividend by any number multiplies (or divides) the quotient by that same number accordingly. $20\div2=?$; $20\div10=?$; $10\div2=?$; $2\div2=?$; $350\div2=?$; $2\times5=?$; $350\div10=?$

60. $80\div4=?$; $4\times4=?$; $80\div16=?$; $100\div5=?$; $5\times10=?$; $5\times4=?$; $100\div50=?$; $100\div20=?$; $12\div3=?$; $3\times2=?$; $12\div6=?$; $12\div4=?$; $4\div2=?$; $12\div2=?$ Thus it is further shown that multiplying the divisor by any number divides the quotient by that number, whilst dividing the divisor by any number multiplies the quotient by that number.

61. An expression indicating division is sometimes called a fraction. This is generally the case whenever the divisor is greater than the dividend. But even where such is not the case, it is called an (improper) fraction provided the dividend be written above, and the divisor below a horizontal line.

62. Bearing in mind, (see multiplication table), that $x\times0=0$, it will be obvious enough that multiplying *any* number by 0 must reduce it to naught; and 0 will therefore necessarily go into *any* number, or even into 0 any desired number of times and still leave the number divided into to carry. And since $x\times1=x$, multiplying or dividing any number by 1 will in no way alter or affect it. But it must not be forgotten that this will be true only when the 1 alone in the first place to the left of the delta, that is, in the units' place constitutes of and by itself the entire divisor or multiplier. That is, when 1 is really one. We can now lay down the following

63. RULE FOR DIVISION.

a. If the divisor has no digits, take whatever you please for quotient and the whole dividend will still be remainder; *β*. If there are no digits in the divisor except one 1, take the dividend for quotient after having removed its ▿ as many places to the right as the divisor has places, if any, to the right of *its* ▿, affixing or erasing 0's if need be;

γ. If the divisor be any power of 10, move the ▿ in the dividend as many places to the left as the divisor has 0's, prefixing or erasing 0's where required, and you get the quotient;

δ. Otherwise place both terms in line, the divisor to the left, and write a curve between them and a horizontal line over the dividend;

ε. If the ▿ in the divisor is not at the extreme right, remove it *to* there;

ζ. Move then the ▿ in the dividend equally far to the right, and no farther, affixing or erasing 0's if necessary;

η. Beginning now at the left, find how many times the divisor the left-hand figure of the dividend will make, (Point 46); and place the figure indicative of such number of times immediately over it;

θ. Multiply the divisor by this figure, placing the product immediately under the figure (or figures) into which the divisor has so been last divided, and subtract it therefrom;

ι. Carry the remainder to the next figure in the dividend to the right—by writing said next figure after it;

κ. Divide, then, into this next figure (so increased by the last remainder as above) and proceed therewith as before;

λ. Proceed thereupon to the then next figure to the right in the dividend in the same manner, then again to the next and the next rightward to the end of the dividend, being careful to copy the ▿ into the quotient just above where it occurs in the dividend now;

μ. Should there be a remainder after the ▿ is reached, or after the last digit to the right of the ▿ is reached, annex 0's (the delta having first been *written*), and continue on the division until it ends without a remainder or the

quotient repeats, unless an answer sufficiently nearly correct is sooner obtained;
ν. Should the left-hand one figure of the dividend fail to contain the divisor, take two or more if needed;
ξ. Finally erase all 0's in the quotient, if any, that are not found to be, at least constructively, between some digit and the delta.

64. ILLUSTRATIVE EXAMPLES.

5∇4326÷2880=?

Operation.

```
      ∇0018863194̇
     ------------
2880)5∇4326000000
     2 880ˣˣˣˣˣˣˣ
     ------
     2 5526
     2 3040
     -------
       24860
       23040
       -----
        18200
        17280
        -----
          9200
          8640
          ----
           5600
           2880
           -----
           27200
           25920
           -----
            12800
            11520
            -----
             1280
```

Explanation.

Find how many of the first, left-hand figure of the divisor the figure of the dividend farthest leftward will make, and, if there 's anything over, carry it as already explained. Said remainder will each time be found by proceeding to multiply and subtract as directed in the rule, but this and the immediate carrying must (or can) here be done mentally. Then ask yourself: Will the next figure of the dividend, thus increased by the said carrying, if carrying there has been, make as many of the next figure of the divisor as the first did of the first? i. e., will the second figure of the divisor, after such carrying, go as often into the thus increased second figure of the dividend as the first figure went or has been suffered to go into the first? Should it so go, then mentally carry what remainder there may be, if any, to the then next figure in the dividend and see will the then next, (now 3d) figure of the divisor also go as often into it as the others went, and so on throughout the divisor. Should they all so go, we place the figure indicating the number of times they went over the right-hand figure of the part of the dividend used, that is, over the figure of the dividend into which, after the proper carrying has been done, we are now dividing. Then we multiply the divisor by it, placing the product under the said used part of the dividend, and subtract it therefrom, carrying the remainder, as before, to the next figure of the dividend. This will needs bring the right-hand figure of the product the first time, (and constructively every time), directly under the figure last put into the quotient. Should the second or any subsequent figure of the divisor fail to go as often as the first went, then the first figure must be let to go a less number of times. But inasmuch as we want to find the highest number of times the divisor will possibly go, we now try it first letting it go only *once* less. Because the highest whole number less than any given whole number is but one less than it. Should it now again fail to go clean across, we must try it once less again, and so on till it *will* go. Should it not even go once, then we divide the first figure of the divisor into the *2* left-hand figures of the dividend, provided this will still leave one apiece for the other figures of the divisor to go into. Otherwise it won't go at all. We must always be careful to fill out the quotient with 0's, if need be, up to the v, whether leftward or rightward. And a pupil must never be suffered to ask himself, *how often* a 2d or subsequent figure of the divisor will go into its corresponding figure (or increased figure) of the dividend, but only, "Will it go as often as the first has been suffered to go?" Nor should he ever be permitted to *guess* the quotient. In the example here given, 2 will go into 5 twice and leave 1 as a remainder. We carry this remainder to the 4, making it 14, (see Point 40). Now, will 8, (the next figure in the divisor), go *as often* into this 14 as the first figure, 2, went into the first figure, 5? No. Therefore 2880 cannot go twice into 5432. But since we want to find the *highest* number of times it *can* go, and since 2 is too high and 1 is the highest (whole) number lower than 2 that there is, we will now let the 2 go but once into the 5. Mentally multiplying the 2 by this 1 so found, we realize that $1 \times 2 = 2$. And subtracting this product mentally from the 5, we find that $5 - 2 = 3$. Carrying this 3 now to the next figure in the dividend renders it 34, the ratio being 10. $10 \times 3 = 30$. $30 + 4 = 34$. Hereupon we ask, Will 8, (the next figure in the divisor), go as often into this 34 as we *now* have suffered the first figure to go into the first figure? But this time we have thus suffered it to go but once. Wherefore the inquiry resolves itself into this: "Will 8 go once into 34?" Yes. And proceeding again as before, we now get 36 for a remainder, which, carried to 3, the *next* figure in the dividend, will make it 263, into which the 3d figure of the divisor will go as often as the first figure was this time suffered to go, that is, once, and leave a remainder of 255. Carrying this remainder to the next figure gives us 2552, and the last figure of the divisor going once into this, we put the 1 into the quotient right over the last or right-hand figure of the (part of the) dividend used or divided into—in this instance the 2, which is, in reality, *the*, by carrying increased figure being divided. The divisor, 2880, went into 543 only 0 times and 543 over, which is carried to the 2. Now, copying up the v, we find 2 spaces between it and the 1 put into the quotient to be filled in with 0's, and we so fill them. It is evident enough without demonstration that no divisor can go into a dividend having less places than itself. 2880 cannot go into 5. Having now reached the ▿, we copy it up. It went 0 times, but as the ▿ will be to the right of this 0, with nothing, no digit, to the left of it, we omit writing it. 0×2880=0, which subtracted from the 5=5. Carrying this remainder to the next figure, 4, gives us 54. But 2880 won't go into 54; or it will again go 0 times. As this 0 is placed over the 4, its place will be between the ▿ and some digit, wherefore we must write it. Now 0×2880 or 2880×0=0, and 54—0=54 to carry. Carried to the next, 3, we get 543, into which our divisor will again go but 0 times. Proceeding as before, and carrying our remainder now again to the then next figure, the 2, we find our divisor will now go into this so increased 2 once. Multiplying the divisor by this 1, we get 2880. Placing the which under the 5432 into which we had just divided, we will naturally get its right-hand figure, 0, under the right-hand figure of the latter, i. e., under the 2, and so under the 1 just put into the quotient. Subtracting, we get 2552 for a remainder. Carrying this to the next figure of the dividend, 6, (by placing said 6 after it), we now find that our 6 will have become 25526. Then dividing into this and proceeding as before, we continue the operation to the end. In the given example it will be seen that the figure in the 10th place in the quotient, the 8th digit, 4, will, or would repeat forever, so we place a "·", the sign of the repetend, over it to indicate this, and stop.

Divide 674▿5 by 11.

Operation.

```
     61▿31̇8̇
   ________
11)674▿5
   66ˣ  ˣ
   --
    14
    11
    --
     35
     33
     --
      20
      11
      --
       90
       88
       --
        2
```

Let us observe that the quotient cannot have as many places, when the terms are prime to each other, to the right of the ▿ or, when there are digits to the right of the ▿, then to the right of the digit in the dividend farthest to the right as the number indicated by the divisor, without either ending or repeating; since if then the same remainder recur a second time, it must repeat, and there cannot be as many *different* remainders as the number indicated by the divisor. Where, however, the dividend ends in a repetend, the quotient will commence to repeat only when the same remainder will be to be carried to the same place in the repetend a second time, the above "rule" notwithstanding.

71040v03÷64=?

Operation.

```
     1110v00046875
   -------------
64)71040v03
   64xxx xx
   --
    70
    64
    --
     64
     64
     -------
       00300
         256
         ---
          440
          384
          ---
           560
           512
           ---
            480
            448
            ---
             320
             320
             ---
               0
```

Or, to be more explicit and illustrative,

```
   01110v00046875
   --------------
64)71040v03000000
   0xxxx xxxxxxxx
   --
   71
   64
   --
    70
    64
    --
     64
     64
     --
      00
       0
      --
       03
        0
       --
        30
         0
        --
        300
        256
        ---
         440
         384
         ---
          560
          512
          ---
           480
           448
           ---
            320
            320
            ---
              0
```

NOTE.—*Marking each figure (with a cross) as "brought down," i. e., as the remainder is carried to it, is not essentially necessary. Nor is annexing the 0's above, though it may, peradventure, also be found convenient.*

$37 \div 15 = ?$

Operation.

$$
\begin{array}{r@{}l}
 & 2\triangledown 4\dot{6} \\ \cline{2-2}
15) & 37\triangledown \\
 & 30 \\ \cline{2-2}
 & \;\;70 \\
 & \;\;60 \\ \cline{2-2}
 & \;\;100 \\
 & \;\;\;\;90 \\ \cline{2-2}
 & \;\;\;\;10
\end{array}
$$

$37\triangledown 6 \div 7 = ?$

Operation.

$$
\begin{array}{r@{}l}
7) & 37\triangledown 6 \\ \cline{2-2}
 & \;\;5\triangledown 3\dot{7}1428\dot{5}
\end{array}
$$

If the divisor has no factors in common with the dividend, and its prime factors are all 2's or 5's, or 2's *and* 5's only, the division will terminate; if it has no 2's nor no 5's as prime factors, the quotient will repeat from the ▿, or, (the dividend having digits to the right of the ▿), from the last, the right-hand one of such digits, unless the dividend ends in a repetend; if it has any 2's or 5's and also at the same time some *other* prime number or numbers as factors, the quotient will also repeat, the repetend, however, commencing farther rightward.

15432v14÷8=?

Operation.

```
  01929v0175
8)15432v1400
  0××××  ××××
  —
  15
   8
  ——
   74
   72
  ———
    23
    16
   ———
     72
     72
    ———
      1
      0
     ———
      14
       8
     ———
       60
       56
      ———
        40
        40
        —
         0
```

Explanation.

Beginning at the left, we find that the first figure we come to in the dividend, 1, will not make the divisor any times, not even once, but only no times. But as it 's plain enough that the 0 representing this (no) number of times will not come between any digit and the v, (the latter being to the right of it, and there being nothing to be placed to the left thereof), we should not write it, or if we do, erase it again. 8×0=0, and this naught subtracted from the 1 (into which we 've just been dividing), leaves 1 for a remainder: 1−0=1. As *this* multiplying and subtracting will have no eventual result, we better omit it. We now carry this "remainder" to the next figure. But since the ratio, locally determined, is 10, this 1, carried to the right, will be 10×1, which is 10. Adding it to the next figure, (5), we get 10+5=15, which, it is plain, will, whenever the ratio is 10, always be just the same as if the number carried were simply written before the next figure. But the selfsame result will be reached if, instead of writing the remainder before the next figure, we write said next figure after the remainder. Thus in carrying 1 to 5, the 5 written after the 1 will make it 15, just the same as when we write the 1 before the 5. Wherefore we carry the remainder to the next figure by writing said next figure after it. Now we must see how many times this 15 will make 8, and place the figure indicative of this number (of times) directly over said 5. Next we multiply the divisor by it, placing the product we thus get under the 5 (made 15) into which we 've last divided, and subtract, etc., etc., as before, continuing thus to the end.

After a little practice a pupil will become able readily to do all this work in his head whenever the divisor is less than 10, putting down the quotient only, as he goes along. He will then be apt to find it as or more convenient to draw the horizontal line *beneath* the dividend instead of over it, and write the quotient below. And since dividing by any power of 10 calls for no work (beyond moving the v), it is only when the divisor is some other number more than 10 that the work above explained and illustrated becomes necessary; and not necessarily even then where the divisor is reducible to prime factors none of which exceeds 7. For 20÷10, for instance, = (20÷2)÷5; 300÷50=(300÷5)÷10, etc., etc., ad sufficiendum. With a needed proper knowledge of the multiplication table it will be as easy (and shorter) to multiply or divide by 9 or by 8 as by their factors, and easier by any power of 10 than by 5 or 2. Thus 6734v2÷900 will easiest be worked as follows: Divide it by 9 and then move the v in the quotient 2 places to the left.

In the following examples run out all divisions to at least the one-millionths' place if they do not terminate sooner, (and as much farther as your teacher may require). Should a quotient commence to repeat before the v is reached, the division must naturally be continued till the entire circle is beyond (to the right of) it, otherwise the answer would be but a meaningless absurdity.

65. EXERCISES.

1. How many Dg of butter @ 3 c. per Hg would pay for a cow worth $37v75?

2. Neptune is four hundred and fifty-nine million Mm from the sun. How long would it take a body to travel from Neptune to the sun at the rate of 51 Cm per h.?

3. The 20th C. commencing with 1 500 000 000 people living on earth if, on an average for the entire C., one is born every v6 s., and 85 die every m., what will be the population of the earth at the end of said 20th C.?

4. A silversmith received 19v136 Cg of silver with which to make 46 tankards. Give the weight of one of these tankards, they being all just alike.

5. A shingle is 1 dm wide and can be laid 1v2 dm to the weather. John Davies's house is 9 m long and the roof, which is double, extends 2 dm beyond the ends. The rafters are 4v32 m long. Required the number of shingles needed to roof it.

6. Alexandre Taché raised 1 dl of flax per ca on 125 Ha of land. What was his crop of flax worth at $4 per Hl?

7. Divide four hundred and seven billion eighteen thousand and ten, *and* 1003 one-hundred-thousandths by one thousand and three ten-thousandths.

8. If there are 700 000 20 people in the 45 United States, how many is that per state on an average?

9. How long would it take a man to walk 1000 Mm, walking 420 Hm every week-day and resting every Sunday?

10. William Edwards slaughtered 15 hogs, which together weighed just 2 T. What was their average weight?

11. 462 + 15v06 + 32 + v12 + 1v6 + 676 + v4 + 18v75 = ?

12. (57 − 3) × (42v5 + 4) = ?

13. 88v8 ÷ 5 = ?

14. 984 ÷ 3 = ?

15. 78 895 465 ÷ 64 = ?

16. (294▿14 ÷ 14) × (671▿55 ÷ 11) = ?

17. (9 770 776▿02 × 4 500) ÷ (1▿011 × 3▿102) = ?

18. (1 ÷ ▿00 000 024) × (▿123 456 789 − ▿12 345 678) = ?

19. $(\overline{1 \div \triangledown 1} \div \triangledown\overline{1 \div 1}) \times \overline{1 \div \triangledown 01} \times (\triangledown 00\ 005 \div \triangledown 000\ 005) = ?$

20. The Rideau Canal is 202 Cm in length and cost $3 860 000. Find the cost per Cm.

21. Divide 70▿04 by 12.

22. Divide sixty octillion one hundred and eight septillion one sextillion eight hundred and fifty-seven quadrillion five hundred and nine trillion twenty-six billion ninety-nine million four hundred and seventy-seven thousand five hundred and eighteen by nine million, *and* seventy-eight one-hundred-nonillionths.

23. $\dfrac{67432358}{4321000} = ?$

24. It is 150 000 000 Cm from here to the sun. If a bullet traveled 46 080 Cm per day, how long would it be crossing the interval?

25. What is the one-tenth part of 11?

26. Divide 4626 by 514.

27. (78 994 632 × 999 998) − (748▿11 × 7▿2) = ?

28. 31 − ▿1 = ?

29. What will 18 pair of gloves cost @ $1▿105 a pair?

30. $\overline{11\ 000 - 9\ 987 \triangledown 623} \times (9 - \triangledown 89) \times (743 + 2 \triangledown 58) = ?$

31. How many d. are there in five hundred and forty billion sixty million one thousand two hundred s.?

32. How many Ha in 274 390 015 ca?

33. Add 629▿31 Mm, 1 400 Cm, 58▿23 Cm, 314 m, 625 Hm, ▿17 634 Cm, 25▿48 Dm, 9▿76 dm, and 11 114 cm together.

34. Multiply 2 wk., 3 d., 19 h., and 13 m. by four hundred and eighteen.

35. How many barrels of sugar, each weighing 11▿7 Mg, can be put aboard a vessel of 451 T burden?

36. 564÷191=?

37. Andy Lee's income last year was $9247v01. He paid out for repairing his house $124, for help $298, and for sundry other items $2008v32. How much did he save?

38. On his farm of 95 Ha, Edward Hand sowed 14 Ha to wheat, 8v35 Ha to potatoes, 75 a to onions, 273 ca to carrots, 41v8 378 Ha to oats, and 11v324 a to rye, leaving the balance for pasture. How much land has he in pasture?

39. Bought 87 Ha, 91 a, and 15 ca of land, for which I paid $175v7 per Ha.

a. What did I pay for it all?
b. How much was that per a?

40. The oldest child in the author's family was born in 1877, the youngest in 1898. What difference in their ages?

41. How many d. will there be in the 20th C.?

42. How many m. will there be in the 22d C.?

43. A certain hotel consumes in the year 1900, daily, 2v5 Cg of bread. How much did it cost @ $1 per Mg?

44. 10v4 876 643÷1v003=?

45. An ant weighing 1 mg can carry 1 cg. How much ought a horse weighing 7 Q carry accordingly?

46. A lady having $311 paid for a hat $14, for a shawl $41, for a dress $62, and for a pair of shoes 63 di. How much money had she then yet left?

47. William Boyd expended $315 for broadcloth of 3 different kinds, taking as many m of one kind as of another. The first piece was $5, the 2d $7, and the 3d $9 per m. How many m did he buy?

48. A certain merchant bought 33 bales of cotton. The first bale weighed 3v4 Q; the next, 9v878 Mg; and the others averaged 100 Cg each. Required their joint weight.

49. He sold the whole lot then for $1077v56. What did he get per Hg for it?

50. To load a car with potatoes I find I have to haul 14 wagon-loads. The car holds 13 s. How much does the wagon hold?

51. Divide 76▿2458 by 620.

52. Joseph Weatherer sold 400 spools of barbed wire @ 3 c. per Cg and found that it amounted to $600. How many cg did one spool weigh?

53. A man willed an estate of $379 146 to his wife and 15 children. To the wife he left one-third of the half thereof, dividing the balance equally amongst the children. How much did a child get?

54. Robb Brothers shipped a consignment of flour @ 50 c. per Mg. They realized $417▿28. How many dg of flour did they ship?

55. (4 764÷5)×91=?

56. Add together 978 m, 20▿95 Dm, 1111▿8 dm, 543▿712 Hm, 1 cm, 5 mm, 2▿76 Cm, ▿023 m, and 76▿2438 Cm; and then divide their sum by 6▿00325.

57. (2+3)×(7▿1÷4)=?

58. 11−(▿5×3)=?

59. How many Townships of 100 Ma each in a rectangular State 50 Mm long and 48 Mm wide?

60. Gold being 19 times as heavy as water, what will a bar of gold a m long, a dm wide, and a cm thick weigh?

61. The divisor is 278▿3, the dividend, 97643. Find the quotient.

62. 43782▿288÷8888=?

63. Paid $10080 for 168 head of cattle. How much was that per head?

64. 95 746 814÷3625▿4=?

65. $\left(\frac{5}{7}+\frac{48-4}{13}\right)\div 33=?$

66. 2785▿0086÷▿0304=?

67. How often will 9▿81 go into fifty-five thousand two hundred and forty-eight, *and* eleven million three hundred and twenty-six thousand and four one-hundred-millionths?

68. How many bullets, each weighing 12 105 mg, can be made from 1 T of lead?

69. How many sheep @ $4▿75 apiece can I buy for three hundred and four dollars?

70. On 11353▿3 Ha of land John O'Leary raised 1 Ms of wheat. How much wheat was that per a?

71. Divide 701▿98704 by 37.

72. How many ^{g} does the sun travel, (the earth turn) in an h.?

73. An insurance company cleared in one year $2 609-391▿4 on 324 148 policies. How much was that on one policy?

74. $(35 \div 4) + (70▿3 \div 6) + (76▿2 \div 91) = ?$

75. A certain little pettifogger and his squire fleeced their victims in one year of $3126, in the next year, of $5100▿75, and in the third year, of $2711▿25. The squire was to get $\frac{1}{3}$ of it, but the other swindled him to the tune of $671▿3. How much did the squire get?

76. $(13 \div 5) - (2 - ▿8\,761) = ?$

77. In a certain realm the people are annually taxed $100 000 000 for public improvements, $600 000 000 to salary the officers for collecting the same and looking after these improvements to see that they are made, $20 000 to be donated to paid-up ex-employees, and $1 876-300▿13 towards the gratuitous support of foreign usurers. How much taxes?

78. 456 sheep yielded 2001 Cg of wool. How much was that per sheep?

79. Sara Conway's sales for January were $2105; for February, $1100; for March, $3275; for April, $4879▿18; for May, $5128▿11; for June, $1700; for July, $888▿78; for August, $1234▿56; for September, $1111▿95; for October, $9000v09; for November, $6271v5; and for December they were $7342v71. What did her sales average per mo.?

80. If a cow will yield 1 Cg of butter per day during June, July, and August, how many cg will that be in all?

81. 48v6 ÷ 54 = ?

82. With $991 Thomas Abélard bought 200 sheep. How much was that per sheep?

83. What number will go 31715 times into 40 000 000-028 040?

66. SOME PRINCIPLES AND PROPERTIES OF NUMBERS.

α′. An even number is always divisible by 2; and if it end in 0, it is also divisible by 10; and if, moreover, the sum of its digits is divisible by 3, it is divisible by 6 also, i. e., divisible leaving no remainder.

β′. If a number end in 5 or 0, it is divisible by 5.

γ′. If the number expressed by its 2 right-hand figures, (barring the ▽), is divisible by 4, it is divisible by 4.

δ′. If the number expressed by its 3 right-hand figures, (not counting the ▽), is divisible by 8, it is divisible by 8.

ε′. If the sum of its digits is divisible by 3, it is divisible by 3; likewise as to 9.

ϛ′. If the sum of the figures standing in the even places is equal to that of those standing in the odd places, the number is divisible by 11; and if this number be composed of, or rather represented by, 4 figures only (besides the ▽), the first and last of which are the same digit and the 2 middle figures 0's; it is also divisible by 7 and by 13.

ζ′. If a number is divisible by any composite number, it is divisible by each and all the factors thereof.

η′. Every number is its own greatest measure.

θ′. A measure of a number is a number which will exactly divide it (leaving no remainder); and such dividend, then, is one of the multiples of such measure.

ι′. An integer is a whole number, and is either prime or composite.

ια′. A prime number is one that has no measure, (exact divisor) other than itself and unity; otherwise it is a composite number.

ιβ′. If 2 or more numbers have the same measure, this is called their common measure, and they are said to be commensurable.

ιγ'. If they have *no* common measure (other than 1), they are prime to each other, incommensurable.

ιδ'. A square number is the product of 2 *equal* factors; and it constitutes the square of either 1 of them.

ιε'. A cube is thus the product of *3* equal factors, and is likewise the cube of (either) one of them, etc., etc.

ιϛ'. A perfect number is exactly equal to the sum of all its lesser measures. It always ends in 6 or 28. The follow8 are the only perfect numbers known; 6; 28; 496; 8 128; 33 550 336; 8 589 869 056; 137 438 691 328; and 2 305 843 008 139 952 128.

ιζ'. The sum of any number of even numbers is an even number.

ιη'. The difference between 2 odd, or between 2 even, numbers will always be an even number.

ιθ'. The sum of an odd number of odd numbers will be an odd number; whilst that of an even number of odd numbers will be an even number.

κ'. The sum of an even and an odd number is ever an odd number; likewise as to their difference.

κα'. If an even number be divisible by an odd number, the quotient will be even.

κβ'. The product of *any* number of factors will be even, if *any*, even one of said factors be even, but if they are *all* odd, the product will be odd.

κγ'. If an odd number measure an even number, it will also measure half of it.

κδ'. Any number that measures 2 other numbers must likewise measure their sum, their difference, and their product.

κε'. Taking *any* 2 unlike numbers, we find that one of them, or their sum, or their difference, will be divisible by 3.

κϛ'. The least measure of any number is always a prime number.

κζ'. Except 2, every prime number is divisible by 4, either when increased by 1 or when so diminished.

κη'. Every prime number, excepting 2 and 3, is divisible by 6, either when increased by 1 or when so diminished.

κθ'. Except 2 and 5, all prime numbers end in 1, 3, 7, or 9.

λ'. Every number is the first power of itself. If multiplied by itself once, (taken twice as a factor), it is in the 2d power, or squared; multiplied once more by itself, (i. e., taken thrice as a factor), and it is cubed, or in the 3d power; multiplied now yet another one time more again by itself, and we have it in the 4th power; and so on ad libitum.

NOTE.—*Any and every number* CAN *be divided into any and every other number, but in these "principles and properties" "divisor" and "divisible" are intended to be understood as being applied only where the division would give an integer only, for the quotient, an exact division.*

An integer or whole number is a plain number wholly to the LEFT *of the* ∇.

67. GREATEST COMMON MEASURE.

When 2 or more numbers are exactly divisible by a given number, this number is their common measure, and if it is the highest number that will do this, it is their *greatest* common measure.

To find the greatest common measure of any 2 numbers, we have the following

68. Rule.

α. Divide the greater by the lesser, then the remainder, if any, into the last divisor, and so on till no remainder is left. The last divisor of all will then be their G.C.M.

β. If now there are *more* than 2 numbers, find the G.C.M. of 2 of them; then take *it* and the 3d of the original numbers and operate on them in the same manner; then, taking this lastly so obtained G.C.M. and the 4th of the given numbers, handle them similarly; and so on to the end. The last final divisor will be the G.C.M. of all.

69. Illustrative Examples.

Find the G.C.M. of 296 and 407.

Operation.

	2, 2	1, 1
111	296	407
	222	296
37	74	111
	74	74
	0	37

Note. — *As we 're not after quotients in this operation, we are indifferent as to where, if anywhere, we put them, for they 're not for further use.*

Answer, 37.

Find the G.C.M. of 1326, 3094, and 4420.

Operation.

	3	2
442	1326	3094
	1326	2652
	0	442

	1 0
442	4420
	442$^{\times}$
	0
	0
	0

Answer, 442.

Find the Greatest Common Measure of 468, 922, and 375.

Operation.

	2, 1	3, 3, 2, 1
454	468	922
	454	468
6	14	454
	12	42$^{\times}$
	2	34
		28
		6
		6
		0

	2	1 8 7
1	2	375
	2	2$^{\times\times}$
	0	17
		16
		15
		14
		1

Answer, 1.

70. EXERCISES.

1. Find the G.C.M. of 110, 140, 680, 770, and 50.
2. Find the G.C.M. of 204, 1190, 1445, and 2006.
3. Find the G.C.M. of 7007, 56321, 101, and 99.
4. Find the G.C.M. of 56, 84, 140, and 168.
5. Find the G.C.M. of 241920, 380160, 69120, and 103680.
6. Find the G.C.M. of 10800, 28040, and 2160.

71. LEAST COMMON MULTIPLE.

A number is a common multiple of 2 or more (other) numbers when each and every one of them will exactly measure or divide it.

It is their *least* common multiple if it happen to be the *lowest* number that they all will so divide.

If all of them will so measure one of them, then that one will be their l.c.m.

Let it be here observed that a dividend will contain a proposed divisor an exact number of times, when it contains as factors *every* factor of that divisor. Wherefore the question of finding the l.c.m. of 2 or more given numbers is reduced to that of finding a number which shall contain as its factors all the prime factors of each and every of the numbers in question and nothing else. If the said numbers in question have no prime factors in common, then their product will be their l.c.m.

72.

EXAMPLE.

Find the l.c.m. of 8, 9, 10, 12, 25, 32, 75, and 80.

Operation.

$$
\begin{array}{l}
\cancel{2})\underline{8, 9, 10, 12, 25, 32, 75, 80}\\
\cancel{2})\underline{4, 9, \;\; 5, \;\; 6, 25, 16, 75, 40}\\
\cancel{2})\underline{2, 9, \;\; 5, \;\; 3, 25, \;\; 8, 75, 20}\\
\cancel{2})\underline{1; 9, \;\; 5, \;\; 3, 25, \;\; 4, 75, 10}\\
\cancel{3})\underline{1, 9, \;\; 5, \;\; 3, 25, \;\; 2, 75, \;\; 5}\\
\cancel{5})\underline{1, 3, \;\; 5, \;\; 1, 25, \;\; 2, 25, \;\; 5}\\
\cancel{5})\underline{1, 3, \;\; 1, \;\; 1, \;\; 5, \;\; 2, \;\; 5, \;\; 1}\\
1, \cancel{3}, \;\; 1, \;\; 1, \;\; 1, \;\; 2, \;\; 1, \;\; 1
\end{array}
$$

$$
\begin{array}{r}
3\\ \hline
6\\
5\\ \hline
30\\
5\\ \hline
150\\
3\\ \hline
450\\
2\\ \hline
900\\
2\\ \hline
1800\\
2\\ \hline
3600\\
2\\ \hline
7200, \text{ l.c.m.}
\end{array}
$$

73. RULE.

α. Write the given numbers in a horizontal line, with a curve before it and a line beneath it;

β. Divide now the several numbers by the lowest number other than 1 that will exactly measure any 2 or more of them, placing the quotients and those of the numbers which it will not so measure below;

γ. Now make a curve before the line of numbers thus written below, and a line under it, and divide by the lowest number (other than 1) that will measure at least 2 of these;

δ. Continue this as far as possible, using, however, whole numbers only for divisors;

ε. Finally multiply together all these divisors together with all the numbers now in the lowest row or line.

74. EXERCISES.

1. What is the least common multiple of the 9 digits?
2. Find the l.c.m. of 6, 9, 12, 15, 18, 21, 27, and 30.
3. Find the l.c.m. of 670, 100, 335, 25, and 440.
4. Find the l.c.m. of 8, 10, 18, 27, 36, 44, and 398.
5. What is the l.c.m. of 12, 20, 24, 14, 21, 3, 2, and 63?
6. Find the l.c.m. of 18, 12, 36, 39, 216, 234, 15, and 70.
7. Find the l.c.m. of 16, 24, 20, 18, 63, 64, 14, and 54.
8. 27, 45, 16, 60, 88, 96, 81, and 49 have what for their l.c.m.?
9. Find the l.c.m. of 120, 40, 39, 65, 88, 16, and 13.
10. What is the l.c.m. of 10, 12, 16, 20, 24, 30, 48, and 6?
11. What is the l.c.m. of 56, 64, 300, 200, and 75?
12. Find the l.c.m. of 150, 50, 60, 20, 60, 15, and 63.
13. Find the l.c.m. of 165, 210, 12, 132, and 144.
14. Find the l.c.m. of 60, 96, 1728, 144, and 18.
15. Find the l.c.m. of 1972, 300, 1000, and 7.
16. Find the l.c.m. of 33, 121, 77, 6006, and 45.
17. Find the l.c.m. of 4004, 9, 103, and 301.
18. Find the l.c.m. of 407, 704, and 200.
19. Find the l.c.m. of 99, 88, 21, 876, 687, and 78.

75. CIRCULATING DECIMALS.

By this time the learner will have noticed that many of the plain numbers which end in a repetend and are known as repeating or circulating decimals cannot always be conveniently handled as desired or required; i.e., not always in that shape. Wherefore it sometimes becomes necessary to transform them first into fractions or mixed numbers. Of such numbers the part which repeats indefinitely is called the repetend or circle. It is also styled the infinite part of the decimal, i.e., of that part of the number to the right of the $\triangledown$, in contradistinction to the rest of said part, which "rest" is called the finite part, (of the said decimal, namely).

If the repetend commences *immediately* after the $\triangledown$, it is called a *pure* repetend, otherwise a *mixed* repetend.

When 2 or more circulating decimals commence to repeat simultaneously, that is, at the same distance from the $\triangledown$, they are said to be similar; and if they all end simultaneously, they are said to be coterminous. (It cannot be necessary to here remark that "distance" in this connection means "number of places from"). Thus: $4\triangledown5\dot{6}0\dot{2}$, $\triangledown0\dot{3}1\dot{5}$, and $1\triangledown8\dot{6}1\dot{5}$ are as well similar as also coterminous.

A little reflection will reveal the fact that, since $\triangledown\dot{8}$, for instance, means that the 8 is to be considered as repeating indefinitely, $\triangledown\dot{8}=\triangledown8\dot{8},=\triangledown88\dot{8}8\dot{8},=\triangledown\dot{8}8\dot{8}$, etc.; for, if these be all run out as indicated, they will very soon all reveal their mutual identity. It must therefore be borne in mind that any pure repetend can be changed into a mixed repetend, or any repetend made to commence to repeat farther to the right without altering its value.

$3\triangledown2\dot{7}6\dot{1}=3\triangledown2761\dot{7}6176\dot{1},=3\triangledown2\dot{7}6176176\dot{1},=3\triangledown276\dot{1}7\dot{6},=3\triangledown27\dot{6}1\dot{7}$. But it must not be forgotten that, if the number of places given to the repetend in any of these extensions be not identical with, or a multiple of, the number indicating the number of places in the original repetend, the equality will at once be destroyed, unless there be but the one figure in the original circle. Thus, if $\triangledown\dot{4}\dot{7}$ and $\triangledown 4\dot{7}4\dot{7}$ be extended, they will vary; whereas $\triangledown\dot{4}\dot{7}$ and $\triangledown 4\dot{7}\dot{4}$ will not; neither will $\triangledown\dot{8}8\dot{8}$ and $\triangledown 8\dot{8}88\dot{8}$ vary.

Hence it will readily be perceived that all repetends can be made to be both similar and coterminous with one another without altering their value.

76. This is done as Follows.

α. Extend all given repetends until they all commence as far to the right as the one originally commencing farthest to the right, and no farther, being careful at the same time to continue each extension until it again has just as many places as it originally had;

β. Then set down in a line the several numbers expressive of the number respectively of places found in each of the (original) repetends or circles, and find their l.c.m.;

γ. Again extend the repetends till each and every one of them has as many places (in the circle) as the number expressed by this l.c.m.

77. Illustrative Examples.

Make $1\triangledown 23\dot{4}$, $\triangledown\dot{7}\dot{1}$, 42, $\triangledown 0\dot{5}46\dot{8}$, and $\triangledown\dot{6}00\dot{8}$ similar and coterminous.

Operation.

$$\left.\begin{array}{r} 1\triangledown 23\dot{4} \\ \triangledown\dot{7}\dot{1} \\ 42\triangledown \\ \triangledown 0\dot{5}46\dot{8} \\ \triangledown\dot{6}00\dot{8} \end{array}\right\} = \left.\begin{array}{r} 1\triangledown 23\dot{4} \\ \triangledown 71\dot{7}\dot{1} \\ \triangledown 05\dot{4}68\dot{5} \\ 42\triangledown 00\dot{0} \\ \triangledown 60\dot{0}86\dot{0} \end{array}\right\} = \begin{array}{r} 1\triangledown 23\dot{4}44\dot{4} \\ \triangledown 71\dot{7}17\dot{1} \\ \triangledown 05\dot{4}68\dot{5} \\ 42\triangledown 00\dot{0}00\dot{0} \\ \triangledown 60\dot{0}86\dot{0} \end{array}$$

$$\begin{array}{l} \cancel{2})\underline{1,\ 2,\ 4,\ 0,\ 4} \\ \cancel{2})\underline{1,\ 1,\ 2,\ 0,\ 2} \\ \underline{}1,\ 1,\ 1,\ 0,\ 1 \\ 4 \end{array}$$

NOTE.— *The row of 0's in the above had better be left out.*

Make $\triangledown 001\dot{0}203\dot{0}$, $4\triangledown 30\dot{4}968\dot{7}$, and $4\triangledown 8712$ similar and coterminous.

Operation.

$$\left.\begin{array}{r} \triangledown 001\dot{0}203\dot{0} \\ 4\triangledown 30\dot{4}968\dot{7} \\ 4\triangledown 8712 \end{array}\right\} = \begin{array}{l} 4\triangledown 8712 \\ \ \triangledown 0010\dot{2}030\dot{0} \\ 4\triangledown 3049\dot{6}874\dot{9} \end{array}$$

78. TO ADD OR SUBTRACT NUMBERS TERMINATING IN REPETENDS.

α. Make them all similar and coterminous and place them one above another so, that the $\triangledown$ will be in column;

β. Commencing at the right, add or subtract just as in other plain numbers, noting first, however, whether or not there 'd be anything to carry to the first column were the repetends all extended a few places farther, and if so carrying accordingly;

γ. Copy down the delta and also the sign or signs of the repetend just beneath where they are now found in the terms above;

δ. If there be 0's on the extreme right in the remainder or sum, these must now be retained so long as any *digit* is left in the repetend thereof, since they are still constructively between a digit and the $\triangledown$;

ϵ. If *all* the figures in the resulting repetend (of the sum or remainder) be 9's, they should be stricken out and 1 carried to the next place leftward instead;

ζ. Finally make the repetend in the answer as short and bring it as far leftward as possible.

79. EXERCISES.

1. Add $48\triangledown\dot{7}\dot{6}$, $8\triangledown 4\dot{0}02\dot{1}$, $\triangledown\dot{3}2343\dot{5}$, and $12\triangledown 1432$ together.
2. $89\triangledown 9\dot{7}9\dot{0} + \triangledown 01284\dot{3} + \triangledown 101213 + 4\triangledown 24 + (5\triangledown 87 - \triangledown 6) = ?$
3. $9\triangledown 6\dot{3}2\dot{7} + 91\triangledown 434 + 71\triangledown 2\dot{0}72\dot{8} + \triangledown 001347321\dot{2}\dot{1} = ?$

4. 8▿8888 + 66▿6 + 8765 + 32▿4̇000̇ + ▿000 005̇ = ?

5. 56▿4 680 00̇0 987̇ − (55▿558 888 8̇12̇ + ▿0 0̇92̇) = ?

6. From 1▿090 880 162 354̇ take ▿987 654 321.

7. 9 + 5▿2 + 4▿7̇3̇ + 88▿1 03̇5 763̇ + ▿0 00̇1 000̇ = ?

8. 1▿275̇ 433̇ − (▿08 076 015 − ▿00 798 764 3̇26̇) = ?

9. (76▿2̇5̇ + 32▿1) − (1▿87̇8 685̇ + 14▿15 161 718 192̇) = ?

10. From one, *and* ninety-nine one-hundred-billionths take ▿34̇5̇.

11. 729▿4̇230̇ + 793▿261 + 793▿2̇61̇ + 1▿2 7̇54̇ + 1▿91̇ = ?

12. From nine, *and* seven hundred and twenty-seven billionths take ▿48 474̇ 652̇.

13. 1▿5 + 1▿5̇ + 3▿0̇9̇ + 9▿9̇0̇ + 88▿9̇8̇ + 7▿0 01̇0̇ = ?

14. (1673▿48̇ 305̇ − 1673▿48 305) + (1 − ▿9̇) + (44 + 5▿5) = ?

80. α′. Any repetend is multiplied by any power of 10 just as any other number is, by merely moving the ▿, but if this bring it to the right of the place where the repetend commenced, the repetend must, of course, be extended till the complete circle, (having as many places as it originally had), is again *all* to the right of the ▿;

β′. A repetend is also readily multiplied by any single digit which is not itself a repetend just as any other plain number is multiplied. But here again whatever there would be to carry to the first product, were the repetend extended a few places farther, must be so carried; and the sign or signs of the repetend must also be copied down, as in addition and subtraction;

γ′. Whenever one of the terms is a composite number having no prime factors higher than 7, the multiplication can best be effected by these, even although the other term terminate in a repetend;

δ′. Any repetend can be divided by any number not itself terminating in a circle;

ε′. In all other cases of multiplication and division of repetends both terms must first be converted into (vulgar)

FRACTIONS. (*Frangere*, to break.)

81. It is an essential property of the Arabic, (more correctly, Hindoostanic) system of notation that, in plain numbers, any digit wherever placed, even although itself the lowest of the digits in value, will still, owing to its *position*, represent a greater number than all that can be written properly to the right of it. The only exception to this is where *all* the figures in the repetend are 9's and 9's only, in which case the entire repetend is just exactly equal to 1 in the first next place leftward.

82. Since the radix is uniformly 10, and since removing the ▽ to the right must, therefore, necessarily multiply a plain number by 10 once for every place it is so moved, and since moving it back again divides it by 10 in the same way thus restoring it to what it was at first inasmuch as multiplying a number by any other number and then dividing the product by that same other number does not eventually alter its value, it will be seen that if the ▽ be removed to the extreme right of any plain number and the said number be then divided by the power of 10 requiring 10 as factor as many times as said ▽ will have so been removed places, its value will not be altered or affected thereby. Bearing in mind also that dividing a number by 1 cannot alter its value, it will now be understood that *any* plain number can be changed into a fraction. Thus:

$$78\triangledown=\frac{78\triangledown}{1\triangledown}$$

$$9=\frac{9}{1}$$

$$100=\frac{100}{1}$$

$$1=\frac{1}{1}$$

$$83=\frac{83}{1}$$

$$\triangledown 5=\frac{5\triangledown}{10\triangledown}$$

$$\triangledown 103=\frac{103\triangledown}{1000\triangledown}$$

$$\triangledown 0087=\frac{87\triangledown}{10000\triangledown}$$

$$\triangledown 602=\frac{602}{1000}$$

$$\triangledown 23=\frac{23\triangledown}{100\triangledown}\text{ or }\frac{23}{100}$$

$$9\triangledown 56=\frac{9\triangledown}{1\triangledown}+\frac{56\triangledown}{100\triangledown}$$

$$82\triangledown 1=\frac{82}{1}+\frac{1\triangledown}{10\triangledown}$$

$$5830\triangledown 02=\frac{5830}{1}+\frac{2\triangledown}{100\triangledown}$$

Or, in the last 3, the integral part, (that to the left of the ▽—see Point 16), can be left unchanged, and that to the right, (*called* "decimal"), written in fractional form, making mixed numbers of them, thus, $9\frac{56}{100}$, $82\frac{1}{10}$, $5830\frac{2}{100}$. Now, keeping in view the fact that multiplying *both* the divisor and the dividend (of a fraction) by the same number will not affect the quotient, we cannot fail to see that, if we multiply both terms of the above fraction $\frac{9}{1}$ by 100 we will have $\frac{900}{100}$, equivalent to $\frac{9}{1}$. And as this was to be added to the $\frac{56}{100}$, we find that we get 900 one-hundredths to be added to 56 one-hundredths. Or $(900\div100)+(56\div100)=\frac{900}{100}+\frac{56}{100},=\frac{956}{100}$, for 900 and $56=956$. Likewise $\frac{82\times10}{1\times10}=\frac{820}{10}$, which, when added to $\frac{1}{10}=\frac{821}{10}$. So also $\frac{5830}{1}+\frac{2}{100}$ will become $\frac{583002}{100}$.

From these and like illustrations we gather that,

83. All that is necessary to convert a plain non-repeating number into a fraction is:

α. If it is *all* to the *left* of the ▽, draw a line beneath it and subscribe a 1;

β. If it has digits on *both* sides of the ▽, erase the ▽ and draw a line beneath, subscribing a 1 followed by as many 0's as the number had places to the right of the ▽;

γ. If the entire number is to the right of the ▽, draw a line beneath it, subscribing a 1 with as many 0's to the right of it as the number had places. Then erase the ▽, and also all 0's, if any, to the left of the most leftward digit therein.

The resulting and all other fractions are again restored to the plain numbers out of which they were manufactured by

Simply working out the division indicated; and that whether the divisor is 1, some power of 10, or any other number.

84. EXERCISES.

1. Change 6∇72 to a fraction.
2. Change 34∇ to a fraction.
3. Change 83 to a fraction.
4. Change 3∇8015 to a fraction.
5. Change 62∇3 to a fraction.
6. Change one, *and* two thousand and five ten-thousandths to a fraction.
7. Change 5∇9098 to a fraction.
8. Change 87∇1 to a fraction.
9. Change 78∇1 to a fraction.
10. Change ∇004 to a fraction.
11. Change ∇01004 to a fraction.
12. $\frac{1}{10}$ is equal to what expressed as a plain number?
13. What will $\frac{7}{10}$ be when the division indicated is worked out? i. e., in what form will it be?
14. What will $\frac{75}{100}$ be when worked out?
15. What will $\frac{8}{4}$ be when worked out?
16. What will $\frac{18}{1}$ be when worked out?
17. What will $\frac{53}{10}$ be when worked out?
18. What will $\frac{11}{10}$ be when worked out?
19. What will $\frac{108}{100}$ be when worked out?
20. What will $\frac{2057}{10}$ be when worked out?

85. But $\frac{2}{7}=2\div7,=\nabla\dot{2}85\,71\dot{4}$; whereas $285714\div1000000$ is *not*$=\frac{2}{7}$. But $285\,714\div999\,999$ *is*$=\frac{2}{7}$. $\frac{1}{3}=1\div3,=\nabla\dot{3}$.

It will further be noticed that $\frac{1}{9}$ is $=\nabla\dot{1}$; $\frac{1}{13}=\nabla\dot{0}7692\dot{3}$; $\frac{3}{11}=3\div11,=\nabla\dot{2}\dot{7},=\frac{27}{99},=\frac{3}{11}$; $\frac{8}{9}=\nabla\dot{8}$, which is the quotient obtained by dividing 9 into 8; $\nabla\dot{0}\dot{5}=\frac{5}{99}$; $\frac{1}{999}=\nabla\dot{0}0\dot{1}$; $\nabla\dot{9}=\frac{9}{9},=1$; $\nabla\dot{9}9\dot{9}=\frac{999}{999},=1$; $\nabla\dot{9}999\,999\,99\dot{9}=1\nabla$; $\nabla99\dot{9}=\nabla\dot{9},=1$. Wherefore

86. TO REDUCE A PURE REPETEND TO ITS EQUIVALENT FRACTION,

a. Draw a line beneath it and subscribe as many 9's as it has places; the repetend really having 9 for *its* ratio.

β. Erase the sign or signs of the repetend and all 0's, if any there be, between its most leftward digit and the ∇, and then erase the ∇ also. This applies though only to numbers that are wholly to the right of the ∇;

γ. If the number have digits or a digit to the left of the ∇, treat that part of it which is to the right as above and then add to the dividend of your fraction the product of the divisor by said part so to the left.

87. It is now evident enough that, whenever a pure repetend has its *every* place occupied by a 9, its quotient, after it has been changed into a fraction, will invariably be exactly 1, nor more, nor less. Hence such a repetend is equal to 1 in the next place leftward, and so ought to, or at least *can*, be erased and 1 carried in lieu thereof as already intimated in Points 81 and 85.

88. $1\nabla 1\dot{1}=\frac{1}{1}$, (i. e., 1 divided by 1), $+\frac{1}{10}$, (i. e., 1 divided by 10, or 1 one-tenth of 1), $+\frac{1}{9}$ of $\frac{1}{10}$ of 1. But $\frac{1}{9}$ of $\frac{1}{10}$ of 1 is, as illustrations above adequately demonstrate, equal to $\frac{1}{90}$ of 1. (See Point 60.) Wherefore $1\nabla 1\dot{1}=\frac{1}{1}+\frac{1}{10}+\frac{1}{90}$. Multiplying both terms of the 2d of these fractions by 9 gives us $\frac{1\times 9}{10\times 9}=\frac{9}{90}$. Now $\frac{9}{90}$ and $\frac{1}{90}=\frac{10}{90}$, for $9+1=10$, whether it be dollars, mètre, ninetieths, or anything else. And multiplying the terms of the first fraction both by 90 makes it $\frac{90}{90}$, which, when added to the $\frac{10}{90}$, makes it $\frac{100}{90}$, for $90+10=100$. $\frac{100}{90}=1\frac{10}{90}$, $=1\frac{1}{9}$. And we have seen in Point 58, that thus multiplying both terms of a fraction by the same number will not change it substantially. $\nabla 2\dot{5}\times 10=2\nabla\dot{5}$. $2\nabla\dot{5}=2\frac{5}{9}$, $=\frac{2}{1}+\frac{5}{9}$. Now, $\frac{2\times 9}{1\times 9}=\frac{18}{9}$, and $\frac{18}{9}+\frac{5}{9}=\frac{23}{9}$. But inasmuch as we have multiplied by 10, this is tenfold too high. To get the quotient correct, we must, therefore, reduce this resulting fraction tenfold, that is, divide it by 10, which is accomplished either by multiplying the divisor by 10, or by dividing the dividend by 10. (See Points 59 and 60.) 10 not being a measure of this dividend, we multiply the divisor.

$$\frac{23}{9\times10}=\frac{23}{90}$$

$$\frac{23}{90}=\frac{23}{9}\div10$$

$$\triangledown50\dot{3}\times100=50\triangledown\dot{3}$$

$$50\triangledown\dot{3}=50\tfrac{3}{9},=\frac{50}{1}+\frac{3}{9}$$

$$\frac{50\times9}{1\times9}=\frac{450}{9}$$

$$\frac{450}{9}+\frac{3}{9}=\frac{453}{9}$$

$$\frac{453}{9\times100},\ (\text{i. e., }\frac{453}{9}\div100)=\frac{453}{900}$$

$$6\triangledown710\dot{4}\dot{7}\times1000=6710\triangledown\dot{4}\dot{7}$$

Now,

$$6710\triangledown\dot{4}\dot{7}=6710\tfrac{47}{99},=\frac{6710}{1}+\frac{47}{99}$$

$$\frac{6710\times99}{1\times99}=\frac{664290}{99},\text{ and }\frac{664290}{99}+\frac{47}{99}=\frac{664337}{99}$$

$$\frac{664337}{99}\div1000=\frac{664337}{99000},=6\frac{70337}{99000}$$

89. But the same results in these and all like cases will more readily and with equal correctness be reached by proceeding as follows:

α. Subtract the finite part of the decimal, (i. e., the part of the number to the right of the $\triangledown$ which does not repeat), from the whole of said decimal, that is, from the whole of that part of the number which is to the right of the $\triangledown$;

β. Taking thereupon the remainder thus obtained for a dividend, draw a horizontal line beneath it, subscribing for a divisor a number expressed by as many 9's as there were originally places in the circle with as many 0's to the right of them as there were originally places in the said finite part of the said "decimal";

γ. Multiply whatever of the number was originally to the *left* of the delta by this divisor and add the product to the dividend already found to complete it.

90. EXERCISES.

1. Change ∇6̇1̇ to its equivalent (vulgar) fraction.
2. Change ∇4̇89̇ to its equivalent fraction in its lowest terms.
3. Change ∇8̇08̇ to its equivalent fraction.
4. Change ∇0̇0040̇ to its equivalent fraction.
5. Change ∇0̇032̇ to its equivalent fraction.
6. Change ∇0̇1̇ to its equivalent fraction.
7. Change ∇4̇ to its equivalent fraction.
8. Change ∇4̇94̇ to its equivalent fraction.
9. Change ∇1̇099̇ to its equivalent fraction.
10. Change 12∇1̇2̇ to its equivalent (improper) fraction.
11. Change ∇362̇0̇ to its equivalent fraction.
12. Change ∇72̇05̇ to its equivalent fraction.
13. Change ∇15161̇ to its equivalent fraction.
14. Change ∇9̇0̇ to its equivalent fraction.
15. Change ∇0̇70̇ to its equivalent fraction.
16. Change ∇26̇489̇ to its equivalent fraction.
17. Change 5∇55̇ to its equivalent fraction in its lowest terms.
18. Change 20∇0̇2̇ to its equivalent fraction.
19. Change 11∇111̇1̇ to its equivalent fraction in its lowest terms.
20. Change ∇987654312̇ to its equivalent fraction.
21. Change ∇1 to its equivalent fraction.
22. Change ∇760̇1̇ to its equivalent fraction.
23. Change 276 to its equivalent fraction.
24. Change 56∇4 to its equivalent fraction.
25. Change ∇9̇9̇ to its equivalent fraction in its lowest terms.
26. Change 123∇ into its equivalent fraction.
27. Change 708∇910̇ into its equivalent fraction.

91. A mixed number is one made up of a whole number and a fraction. Thus $17\frac{2}{7}$, $1\frac{14}{15}$, $908766\frac{1}{4}$, $3\frac{5}{9}$, $25\frac{8}{11}$, $1\frac{100}{203}$, $6\frac{75}{100}$, $32\frac{8}{9}$, $5\frac{3758}{10000}$, $8\frac{176}{9999}$, $2\frac{4}{100}$, $11\frac{11}{900}$ are mixed numbers when written in this style. In all mixed numbers the right-hand figure of the whole number ought ever to be in the units' place. But in any case the fraction expresses such a part of, so much of, 1 in the place of said right-hand figure of the whole number. So we have plain numbers, mixed numbers, and fractions.

92. To change a mixed number into its equivalent fraction,

Add to the dividend of the fraction the product of its divisor by the whole number, and then erase the said whole number.

NOTE.— *The result will necessarily always be an improper fraction.* (See Points 83 and 89.)

93. EXERCISES.

Change the following mixed numbers into their equivalent fractions:

1. $76\frac{3}{11}$;
2. $14\frac{14}{19}$;
3. $562\frac{1}{2}$;
4. $21\frac{11}{12}$;
5. $1\frac{1}{14}$;
6. $88\frac{8}{19}$;
7. $528\frac{9}{10}$;
8. $63214\frac{9}{13}$;
9. $1\frac{1}{2}$;
10. $21\frac{7}{8}$;
11. $50\frac{1}{5}$;
12. $42\frac{3}{4}$;
13. $17\frac{7}{11}$;
14. $29\frac{8}{11}$;
15. $8\frac{1}{5}$;
16. $9\frac{4}{9}$;
17. $563\frac{4}{5}$;
18. $808\frac{3}{4}$;
19. $1415\frac{1}{6}$;
20. $254\frac{1}{2}$;
21. $575\frac{3}{8}$;
22. $848\frac{1}{7}$;
23. $971\frac{5}{9}$;
24. $87\frac{1}{3}$;
25. $2728\frac{1}{2}$;
26. $1510\frac{5}{11}$;
27. $545\frac{8}{13}$;
28. $203\frac{4}{5}$;
29. $4\frac{327}{484}$;
30. $92\frac{11}{101}$;
31. $888\frac{8}{15}$;
32. $97\frac{3}{17}$;
33. $627\frac{61}{712}$;
34. $767\frac{2}{25}$;
35. $676\frac{2}{71}$.

94. To now change these or any other improper fractions back again to their equivalent whole or mixed numbers,

Work out the division indicated, stopping at the v.

The quotient thus obtained will be the whole number, and if there be a remainder, *it* will be the dividend of the fractional part, of which the old divisor will again be the divisor.

95. ILLUSTRATIVE EXAMPLES.

Change $\frac{27}{3}$ to its equivalent whole or mixed number.

Operation.

$27 \div 3 = 9.$

Likewise change $\frac{29}{2}$, $\frac{17}{16}$, $\frac{273}{3}$, and $\frac{95}{1}$.

Operations.

$29 \div 2 = 14\frac{1}{2}$. $17 \div 16 = 1\frac{1}{16}$. $273 \div 3 = 91$. $95 \div 1 = 95$.

96. EXERCISES.

Change in like manner,

1. $\frac{31}{9}$; 3. $\frac{82}{2}$; 5. $\frac{888}{88}$; 7. $\frac{189}{11}$; 9. $\frac{208}{1}$;
2. $\frac{706}{5}$; 4. $\frac{61784}{10}$; 6. $\frac{99}{8}$; 8. $\frac{141}{8}$; 10. $\frac{742}{100}$.

97. A fraction is in its lowest terms when these are prime to each other.

If they are not, they can be made so by dividing both by their G.C.M., the way to find which has already been shown in Point 68.

98. EXERCISES.

Reduce to their lowest terms the following:—

1. $\frac{77}{49}$; 8. $\frac{36}{156}$; 15. $\frac{7}{17}$; 22. $\frac{41}{14}$; 29. $\frac{828}{8481}$; 36. $\frac{212}{6167}$;
2. $\frac{1048}{10000}$; 9. $\frac{11111}{2002}$; 16. $\frac{6524}{5654}$; 23. $\frac{576}{276}$; 30. $\frac{212}{1719}$; 37. $\frac{35}{55}$;
3. $\frac{976}{5}$; 10. $\frac{69963}{33963}$; 17. $\frac{888}{3773}$; 24. $\frac{676}{7176}$; 31. $\frac{31}{471}$; 38. $\frac{101}{2525}$;
4. $\frac{42}{6006}$; 11. $\frac{18}{100}$; 18. $\frac{54}{612}$; 25. $\frac{5762}{1815}$; 32. $\frac{545}{414}$; 39. $\frac{141}{414}$;
5. $\frac{13}{26}$; 12. $\frac{25}{555}$; 19. $\frac{32}{781}$; 26. $\frac{4142}{47484}$; 33. $\frac{565}{2127}$; 40. $\frac{28}{828}$;
6. $\frac{63}{64}$; 13. $\frac{40}{48}$; 20. $\frac{16}{6}$; 27. $\frac{3281}{2127}$; 34. $\frac{1212}{1719}$; 41. $\frac{9194}{42004}$;
7. $\frac{64}{1000}$; 14. $\frac{90}{819}$; 21. $\frac{187}{666}$; 28. $\frac{1215}{1050}$; 35. $\frac{329}{8786}$; 42. $\frac{61}{16104}$.

99. $\frac{3}{\text{sheep}} + \frac{2}{\text{sheep}} = \frac{5}{\text{sheep}}$. $\frac{2}{\text{dollars}} + \frac{5}{\text{dimes}} = ?$

$5+2=7$, but this 7 will be neither dollars nor dimes. As with plain numbers, so with fractions, only *like* numbers can be added—or subtracted. The dollars, however, might be reduced to dimes, or vice versa, and then added.

NOTE.—*The divisor in a fraction serves also to* NAME *it.*

If 2 or more fractions have not the same divisor (name) in common, they must be changed into equivalent fractions all having the same divisor in common, before they can be added or subtracted.

100. TO DO THIS,

a. Find the l.c.m. of all the divisors. (Point 73.)

This will be the least common divisor possible, and the least will be the most preferable one to be sought;

β. Divide this l.c.m. by the divisor of the first fraction, multiplying the quotient so obtained by its dividend for a new dividend;

γ. Do the same in succession with all the other fractions.

NOTE.—*This is in effect but multiplying both terms each time by the same number.*

ILLUSTRATIVE EXAMPLE.

101. Let it be required to change $\frac{7}{8}$, $\frac{3}{5}$, $\frac{9}{10}$, $\frac{11}{15}$, $\frac{10}{20}$, and $\frac{7}{16}$ to equivalent fractions respectively all having a common divisor.

Operation.

$$\cancel{2})\underline{\tfrac{7}{8},\ \tfrac{3}{5},\ \tfrac{9}{10},\ \tfrac{11}{15},\ \tfrac{10}{20},\ \tfrac{7}{16} = \tfrac{210}{240},\ \tfrac{144}{240},\ \tfrac{216}{240},\ \tfrac{176}{240},\ \tfrac{120}{240},\ \tfrac{105}{240}.}$$

$$\begin{array}{l}
\cancel{2})\underline{4,\ 5,\ \ 5,\ 15,\ 10,\ \ 8} \\
\cancel{2})\underline{2,\ 5,\ \ 5,\ 15,\ \ 5,\ \ 4} \\
\cancel{5})\underline{1,\ 5,\ \ 5,\ 15,\ \ 5,\ \ 2} \\
\ \ \ \ 1,\ 1,\ \ 1,\ \ \cancel{3},\ \ 1,\ \ 2
\end{array}
\qquad
\begin{array}{r}
2 \\ \underline{3} \\ 6 \\ \underline{5} \\ 30 \\ \underline{2} \\ 60 \\ \underline{2} \\ 120 \\ \underline{2} \\ 240 \text{ l.c.m.}
\end{array}$$

$$\begin{array}{r}
5)\underline{240} \\ 48 \\ \underline{3} \\ 144
\end{array}
\qquad
\begin{array}{r}
15)\underline{240} \\ 16 \\ \underline{16} \\ 176
\end{array}
\qquad
\begin{array}{r}
16)\underline{240} \\ 15 \\ \underline{7} \\ 105
\end{array}$$

$$\begin{array}{r}
8)\underline{240} \\ 30 \\ \underline{7} \\ 210
\end{array}
\qquad
\begin{array}{r}
10)\underline{240} \\ 24 \\ \underline{9} \\ 216
\end{array}
\qquad
\begin{array}{r}
20)\underline{240} \\ 120
\end{array}$$

If now it be desired to add these, all that will be required will be to add the new dividends together, draw a line beneath the sum and subscribe this common namer or divisor: 210+144+216+176+120+105=971, making $\frac{971}{240}$.

For subtraction change the mixed numbers to their equivalent improper fractions first. But for addition add the whole numbers separately, then the fractions separately also, and finally add these 2 sums together, having changed the fraction, if improper, to a mixed number.

Illustrative Examples.

Subtract $\frac{14}{35}$ from $\frac{65}{77}$.

Operation.

$$\cancel{7})\underline{\tfrac{65}{77}-\tfrac{14}{35}=\tfrac{325}{385}-\tfrac{154}{385}=\tfrac{171}{385}}.$$

$$\begin{array}{lr}
\cancel{11} & 5\\
 & 5\\ \cline{2-2}
 & 55\\
 & 7\\ \cline{2-2}
 & 385\text{ l.c.m.}
\end{array}
\qquad
\begin{array}{r}
385\div77=\ \ 5,\text{ and }65\times\ \ 5=325\\
385\div35=11,\text{ and }14\times11=\underline{154}\\
171
\end{array}$$

Add $3\frac{1}{5}$, $\frac{5}{8}$, and $\frac{11}{12}$.

Operation.

$$\begin{array}{l}
\cancel{2})\underline{\tfrac{1}{5}+\tfrac{5}{8}+\tfrac{11}{12}}=\tfrac{24}{120}+\tfrac{75}{120}+\tfrac{110}{120}=\tfrac{209}{120},=1\tfrac{89}{120}.\\
\cancel{2})\underline{5\quad 4\quad 6}\\
\quad\cancel{5}\quad\cancel{2}\quad 3
\end{array}
\qquad
\begin{array}{r}
\\
\underline{3\quad}\\
4\tfrac{89}{120}.
\end{array}$$

$$\begin{array}{l}
3\times2\times5\times2\times2=120\text{ l.c.m.}\\
120\div\ \ 5=24,\text{ and }24\times\ \ 1=\ \ 24\\
120\div\ \ 8=15,\text{ and }15\times\ \ 5=\ \ 75\\
120\div12=10,\text{ and }10\times11=\underline{110}\\
\hfill 209
\end{array}
\qquad
\begin{array}{r}
1\tfrac{89}{120}\\
120)\overline{209}\\
\underline{120}\\
89
\end{array}$$

102. EXERCISES.

In like manner change and add,

1. $\frac{61}{78}$, $\frac{3}{5}$, $\frac{4}{15}$, $\frac{18}{39}$, $\frac{1}{4}$, $\frac{200}{703}$, $\frac{18}{13}$, and $1\frac{5}{11}$;
2. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{9}$;
3. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$;
4. $\frac{9}{18}$, $\frac{15}{16}$, $\frac{1}{5}$, and $\frac{31}{110}$;
5. $\frac{6}{11}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{18}{77}$, and $\frac{19}{33}$;
6. $\frac{3}{5}$, $\frac{7}{10}$, $\frac{6}{25}$, $\frac{11}{30}$, $\frac{13}{45}$, and $\frac{23}{60}$;
7. $\frac{1}{4}$, $\frac{2}{3}$, $\frac{9}{16}$, $\frac{14}{15}$, $\frac{4}{8}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{6}{11}$, $\frac{19}{20}$, $\frac{6}{7}$, and $\frac{29}{35}$;
8. $\frac{8}{15}$, $\frac{9}{11}$, $\frac{21}{141}$, $\frac{52}{145}$, $\frac{6}{7}$, $\frac{19}{21}$, $\frac{27}{93}$, $\frac{60}{75}$, $\frac{2}{3}$, and $\frac{12}{13}$;
9. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{9}$, $\frac{1}{11}$, and $\frac{6}{13}$; and
10. $\frac{8}{12}$, $\frac{8}{15}$, $\frac{8}{17}$, $\frac{8}{19}$, and $\frac{8}{11}$. Then subtract,
11. $\frac{7}{9}+\frac{1}{2}$ from $2\frac{3}{4}-\frac{5}{4}$;
12. 51 from $82\frac{3}{5}-4$;
13. $\frac{6}{15}+\frac{4}{9}$ from $\frac{1}{7}+\frac{8}{9}+\frac{1}{2}$;
14. $\frac{25}{27}-\frac{1}{18}$ from $1\frac{5}{8}+\frac{3}{5}$;
15. $\frac{81}{100}$ from $\nabla 9$;
16. $7\frac{1}{5}$ from $25-4\frac{2}{9}$; and
17. $1\frac{1}{4}$ from $7-\frac{4}{5}$.

103. Five sixes means 5 of 6, or 6 taken 5 times; in short, 6×5, just as 5 bricks will mean 1 brick repeated 5 times, or brick×5. And this is true of *any* 2 numbers so taken, be they fractions or plain or mixed numbers.

Two of three of four will accordingly mean 3 of 4, (i.e., 4×3), taken twice.

Let it now be required to find out how much $\frac{13}{17}$ of $\frac{7}{11}$ will be.

Operation.

$\frac{13}{17}$ of $\frac{7}{11}$ means 13 times $\frac{1}{17}$ of $\frac{7}{11}$.

If we multiply the divisor of the $\frac{7}{11}$ by 17 we will get $\frac{1}{17}$ of the $\frac{7}{11}$. (See Point 60.) $\frac{1}{17}$ of $\frac{7}{11}=\frac{7}{11\times17},=\frac{7}{187}$; and if we now multiply this result by 13, we will have 13 times $\frac{7}{187}$, that is, $\frac{13}{17}$ of $\frac{7}{11}$. But multiplying the dividend multiplies the quotient, (hence the fraction). (See Point 59.)

So we multiply the dividend by the 13 accordingly. $\frac{7}{11\times17}\times13=\frac{7\times13}{11\times17},=\frac{91}{187},=\frac{13}{17}$ of $\frac{7}{11}$.

104. Wherefore to multiply 2 or more fractions together,

Multiply all the dividends together for a new dividend, and all the divisors together for a new divisor.

105. ILLUSTRATIVE EXAMPLES.

How much is $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{4}{9}$?

Operation.

$\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{4}{9}=\frac{2}{3}\times\frac{5}{7}\times\frac{4}{9},=\frac{40}{189}$; that is, $2\times5\times4=40$, and $3\times7\times9=189$, whence we get the $\frac{40}{189}$.

How much is $\frac{5}{8}$ of $\frac{1}{10}$ of $\frac{4}{5}$ of $\frac{2}{3}$?

Operation.

$\frac{5}{8}$ of $\frac{1}{10}$ of $\frac{4}{5}$ of $\frac{2}{3}=\frac{5}{8}\times\frac{1}{10}\times\frac{4}{5}\times\frac{2}{3},=\frac{40}{1200}$.

But this last result is evidently not in its lowest terms. As 40 is a factor common to both terms, they should both be divided by it to eliminate it. $\frac{40\div40}{1200\div40}=\frac{1}{30}$. The better way will be to first cast this common factor out of the original fractions piecemeal, by cancellation, thus:

$$\frac{\cancel{5}^{1}}{\cancel{8}_{2}}\times\frac{1}{\cancel{10}_{\cancel{2}1}}\times\frac{\cancel{4}^{1}}{5}\times\frac{\cancel{2}^{1}}{3}=\frac{1}{30}$$

EXPLANATION.

5 will go once into one of the dividends, the first, and twice into the second of the divisors; 2 will go once into the 2 thus secured, and once into the last of the given dividends; and 4 will go once into the 3d of the dividends, and twice into the first of said given divisors. Our dividends are now *all* reduced to 1's, whilst the divisors have become 2, 1, 5, and 3 respectively. $1\times1\times1\times1=1$, and $2\times1\times5\times3=30$, which gives us the answer, $\frac{1}{30}$.

106. EXERCISES.

1. How much is $\frac{6}{11}$ of $\frac{4}{7}$ of $\frac{3}{5}$ of $\frac{22}{27}$ of $\frac{35}{16}$ of $\frac{4}{9}$ of $\frac{1}{3}$ of $\frac{45}{66}$?
2. Find the value of $\frac{7}{11}$ of $\frac{4}{5}$ of $\frac{3}{7}$ of $\frac{19}{23}$.
3. Multiply together $6\frac{1}{2}$, $\frac{3}{13}$, $\frac{1}{11}$, $\frac{18}{25}$, $\frac{5}{9}$, 2, and $1\frac{1}{2}$.
4. $\frac{4}{7}$ of $\frac{3}{5}$ of $\frac{6}{11}$ of $\frac{35}{72}$ of $\frac{6}{11}$ of $\frac{77}{35}$ of $\frac{2}{3}$ of $\frac{4}{9}$ of $\frac{6}{7}$ of $\frac{81}{100}$ of $\frac{21}{31}=?$
5. $3\times\frac{1}{4}\times3\frac{1}{4}\times\frac{7}{8}\times\frac{3}{4}\times1\frac{9}{10}\times\frac{2}{5}\times\frac{5}{13}=?$
6. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $11=?$
7. $\frac{8}{17}\times\frac{13}{21}\times\frac{17}{18}\times\frac{7}{19}=?$
8. $\frac{8}{9}\times\frac{9}{10}\times\frac{10}{11}\times\frac{12}{13}\times\frac{15}{17}$ of $\frac{2}{5}\times1\frac{8}{11}\times\frac{3}{4}\times0\times\frac{5}{19}\times\frac{6}{7}\times\frac{3}{14}=?$

107. What part of a d. is $\frac{1}{2}$ a wk.?

As there are 7 d. in a wk., we have but to multiply by 7, the question really being simply: How many d. in so many wk.? $\frac{1}{2}\times7=\frac{1}{2}\times\frac{7}{1},=\frac{7}{2}$. Working out the now indicated division gives us $3\frac{1}{2}$.

$\frac{5}{6}$ of $\frac{1}{2}$ m is how many dm?

Operation.

$\frac{5}{6}$ of $\frac{1}{2}=\frac{5}{6}\times\frac{1}{2}$. As there are 10 dm in a m, we must multiply this, (the mètre) by 10, which gives us

$$\frac{5}{6}\times\frac{1}{2}\times10=\frac{5}{6}\times\frac{1}{\cancel{2}}\times\frac{\cancel{10}^{5}}{1},=\frac{25}{6},=4\frac{1}{6}\text{ dm.}$$

$\frac{3}{11}$ of $2\frac{1}{4}$ mm = how many Mm?

Operation.

$\frac{3}{11}$ of $2\frac{1}{4}=\frac{3}{11}\times\frac{9}{4},=\frac{27}{44}$. Now, since there are 10 000 000 mm in a Mm, we must *divide* this number, $\frac{27}{44}$, by 10 000 000, which is accomplished by multiplying the divisor, 44, by the 10 000 000. (Point 60.) $44\times10\,000\,000=440\,000\,000$. $\frac{27}{44}\div10\,000\,000$, therefore, is $\frac{27}{440000000}$.

$\frac{8}{15}$ of $\frac{7}{8}$ of $\frac{3}{5}$ of 2 T = how many Q?

Operation.

$\frac{8}{15}$ of $\frac{7}{8}$ of $\frac{3}{5}$ of $2=\frac{8}{15}\times\frac{7}{8}\times\frac{3}{5}\times\frac{2}{1}$, equal (when canceled) to $\frac{1}{5}\times\frac{7}{1}\times\frac{1}{5}\times\frac{2}{1},=\frac{14}{25}$. $\frac{14}{25}\times10=\frac{140}{25},=5\frac{15}{25},=5\frac{3}{5}$.

$\frac{1}{11}$ of $7\frac{1}{8}$ Cg = how many g?

Operation.

$$\frac{1}{11}\text{ of }7\frac{1}{8}=\frac{1}{11}\times\frac{57}{8},=\frac{57}{88}.\quad \frac{57}{58}\times1000=\frac{57}{\cancel{88}_{11}}\times\frac{\cancel{1000}^{125}}{1},=\frac{7125}{11},=647\frac{8}{11}.$$

108. To multiply any plain number by a fraction having one for its dividend we can simply *divide* the plain number by the fraction's divisor.

109. ILLUSTRATIONS.

$125\times\frac{1}{5}=\frac{125}{1}\times\frac{1}{5}$, i. e., $\frac{25}{1}\times\frac{1}{1}=\frac{25}{1},=25$. Or $125\div5=25$, equal, in short, to $125\times\frac{1}{5}$.

$2\triangledown7\times\frac{1}{6}=\frac{27}{10}\times\frac{1}{6},=\frac{27}{60}$. Or $2\triangledown7\div6=\triangledown45$.

$80\triangledown0\dot{2}\times\frac{1}{4}=\frac{7202}{90}\times\frac{1}{4},=20\triangledown00\dot{5}$. Or $80\triangledown0\dot{2}\div4=20\triangledown00\dot{5}$.

110. EXERCISES.

1. $100\times\frac{1}{4}=?$
2. $16\times\frac{1}{2}=?$
3. $14\times\frac{1}{4}=?$
4. $710\times\frac{1}{24}=?$
5. $311\times\frac{1}{19}=?$
6. $212\times\frac{1}{6}=?$
7. $151\times\frac{1}{4}=?$
8. $658\times\frac{1}{9}=?$
9. $50\times\frac{1}{8}=?$
10. $21\times\frac{1}{3}=?$
11. $76843\times\frac{1}{11}=?$
12. $6731\times\frac{1}{12}=?$
13. $76267\times\frac{1}{9}=?$
14. $6260\times\frac{1}{2}=?$
15. $3620\times\frac{1}{7}=?$
16. $173\times\frac{1}{14}=?$
17. $2002\times\frac{1}{7}=?$
18. $5885\times\frac{1}{18}=?$
19. $717273\times\frac{1}{3}=?$
20. $30000\times\frac{1}{15}=?$

111. Let it be required to divide $\frac{4}{5}$ by $\frac{3}{7}$.

Operation.

$\frac{3}{7}$ is $\frac{1}{7}$ of 3, just as 3 fishes would be 1 fish taken 3 times, or 1×3, (namely, fishes). $\frac{1}{7}$ of 3 is $\frac{1}{7}\times\frac{3}{1}=\frac{3}{7}$. Now if we divide $\frac{4}{5}$ by 3, the quotient will be sevenfold too small, since the proposed divisor is but $\frac{1}{7}$ of 3, wherefore 3 as a divisor is 7 times too great. $\frac{4}{5}$, as we know, will be divided by 3 if we multiply its divisor, 5 by the 3, thus, $\frac{4}{5\times3}=\frac{4}{15}$, which "quotient," as just explained, is sevenfold too small. It is, therefore, now necessary to multiply it by 7, the which is accomplished by simply so multiplying the dividend. $\frac{4^{\times7}}{15}=\frac{28}{15},=1\frac{13}{15}$. (Point 59.)

112. Therefore, to divide fractions or by fractions,

a. Reduce both terms to simple fractions;

β. Invert the divisor and then proceed just as in multiplication;

γ. If the dividend be a plain number, and the divisor be a fraction having 1 for *its* dividend, multiply the dividend by the divisor of the divisor.

δ. Or, where practicable, first change both terms into plain numbers.

113. ILLUSTRATIVE EXAMPLES.

Divide $\frac{4}{11}$ by $\frac{5}{9}$.

Operation.

$\frac{4}{11}\div\frac{5}{9}=\frac{4}{11}\times\frac{9}{5},=\frac{36}{55}$.

Divide 3 times $\frac{1}{4}$ by 6 times $\frac{1}{7}$.

Operation.

3 times $\frac{1}{4}=3$ of $\frac{1}{4},=\frac{3}{1}$ of $\frac{1}{4},=\frac{3}{1}\times\frac{1}{4},=\frac{3}{4}$; and
6 times $\frac{1}{7}=6$ of $\frac{1}{7},=\frac{6}{1}$ of $\frac{1}{7},=\frac{6}{1}\times\frac{1}{7},=\frac{6}{7}$;

$\frac{3}{4}\div\frac{6}{7}=\frac{\cancel{3}^{1}}{4}\times\frac{7}{\cancel{6}_{2}},=\frac{7}{8}$.

$$\cfrac{\cfrac{3\frac{1}{4}}{\frac{1}{5}}}{\cfrac{7\frac{1}{2}}{\cfrac{3\frac{1}{7}}{\frac{5}{9}}}}=?$$

Operation.

$\cfrac{3\frac{1}{4}}{\frac{1}{5}}=3\frac{1}{4}\div\frac{1}{5}$, i. e., $\frac{13}{4}\div\frac{1}{5},=\frac{18}{4}\times\frac{5}{1},=\frac{65}{4}$;

$\cfrac{3\frac{1}{7}}{\frac{5}{9}}=3\frac{1}{7}\div\frac{5}{9}$, i. e., $\frac{22}{7}\div\frac{5}{9},=\frac{22}{7}\times\frac{9}{5},=\frac{198}{35}$;

$\cfrac{7\frac{1}{2}}{\frac{198}{35}}=7\frac{1}{2}\div\frac{198}{35}$, i. e., $\frac{15}{2}\div\frac{198}{35},=\frac{\cancel{15}^{5}}{2}\times\frac{35}{\cancel{198}_{66}},=\frac{175}{132}$;

$\cfrac{\frac{65}{4}}{\frac{175}{132}}=\frac{65}{4}\div\frac{175}{132},=\frac{\cancel{65}^{13}}{\cancel{4}}\times\frac{\cancel{132}^{33}}{\cancel{175}_{35}},=\frac{429}{35},=12\frac{9}{35}$.

114. EXERCISES.

1. $(\frac{3}{5}$ times $\frac{7}{11})\div(\frac{4}{15}$ of $3)=?$
2. $578\div(\frac{1}{2}$ of $\frac{2}{5}\times\frac{7}{10})=?$
3. $\cfrac{\frac{11}{12}}{\frac{1}{5}}-(4\div7)+(3\times\frac{5}{18})=?$

4. The Niagara River is 6 Mm long, the St. Laurent is 320 Mm long. What part of the length of the St. Laurent is that of the Niagara? I. e., how many times the length of the St. Laurent is the Niagara?

5. What will $6\frac{7}{8}$ Cg of tea cost @ $19\frac{3}{4}$ c. per Hg?

6. Mr. Conway owns an undivided $\frac{1}{8}$ interest in a steamboat. Mr. McLean's share therein is $3\frac{1}{2}$ times $\frac{1}{5}$ of what Mr. Conway's is, and Mr. Scanlon's share is twice as much as Mr. Conway's plus $\frac{1}{3}$ of Mr. McLean's share. How much of the ship will Mr. Conway own after buying out Messrs. McLean and Scanlon?

7. Divide seven Mm, 8 Hm, forty-eight m, and 5 cm, by 14.

8. A wall is to be built 16 m long, 2v8 m high, and 6 dm thick. If each brick with its mortar will fill in a space 2 dm long, 1 dm wide, and 6 cm thick, how many bricks will it require, a doorway 1×2 m being left in the wall?

9. Reduce 82 703 458v3 g to Q.

10. It is 62 Cm from Toronto to Hamilton. How many times will a carriage wheel revolve in making the journey if the circumference of the wheel be 4v15 m?

11. $\dfrac{\frac{2}{7\frac{1}{2}}}{61} = ?$

12. $\dfrac{\frac{\frac{1}{2}}{8}}{15\frac{1}{2}} = ?$

13. $\dfrac{\frac{3}{5}}{\frac{4}{\frac{1}{2}}} = ?$

14. $76254 \div \frac{1}{4} = ?$

15. $19 \div \frac{1}{7} = ?$

16. $5184 \div \frac{1}{11} = ?$

17. $88 \div \frac{1}{18} = ?$

18. $2788 \div \frac{1}{8} = ?$

19. Divide 3 by $\frac{1}{7}$.

20. $\dfrac{\frac{2}{5}}{\frac{5}{8} - \frac{1}{2}} = ?$

21. $8882 \div 76\frac{11}{18} = ?$

22. $\dfrac{\frac{4}{5}}{\frac{5}{6}} = ?$

23. $\frac{9}{14}$ divided by $\frac{14}{99} = ?$

24. $\frac{82}{45} \div 1\frac{1}{8} = ?$

25. $\frac{59}{118} \div \frac{5}{8} = ?$

26. $(\frac{41}{88}$ of $\frac{1}{2}) \div \frac{1}{24} = ?$

27. $\frac{5}{7} \div \frac{7}{10} = ?$

28. What part of the St. Laurent River would the Ottawa, which is 128 Mm long, make?

29. How many times the length of the St. Laurent is the Chaudière, which is 160 Cm long?

30. What part of 12 will $\frac{1}{3}+\frac{11}{10}+\frac{1}{8}+\frac{1}{6}+\frac{1}{12}+\frac{7}{20}$ be?

31. A tree in falling was broken into 2 pieces. $\frac{3}{4}$ of the shorter piece $=\frac{2}{3}$ of the longer. The tree having been 425 dm in length, how long was,

a. The shorter piece?

b. The longer piece?

32. $(\frac{4}{5}\div\overline{3\frac{1}{8}\times\frac{2}{3}\text{ of }\frac{1}{2}})\times\frac{7/8}{4}+1\frac{1}{9}=?$

33. $\dfrac{5\frac{4}{5}-2\frac{1}{8}}{3\frac{3}{4}+\frac{9}{20}}\text{ of }\dfrac{4\frac{1}{2}+5\frac{19}{25}}{4\frac{1}{20}}\text{ of }\dfrac{2\frac{3}{5}+1\frac{2}{3}}{7\frac{19}{24}-2\frac{1}{4}}=?$

34. $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\dfrac{1}{2\frac{1}{2}}+\dfrac{1}{3\frac{1}{4}}+\dfrac{1}{4\frac{1}{2}}}=?$

35. $(\frac{1}{2}+\frac{1}{3})\times(1\frac{1}{3}+2\frac{3}{4})\times(2\frac{1}{14}-1\frac{1}{2})\times(3\frac{1}{4}\div\frac{8}{17})=?$

36. $\frac{2}{3}\div1\frac{5}{7}-\frac{5}{8}\div3\frac{2}{11}=?$

37. $\{(12\frac{5}{6}-8\frac{3}{4})-(1\frac{1}{10}+\frac{8}{15})\}\times4\frac{1}{2}\text{ of }\overline{7\frac{5}{12}-6\frac{1}{2}}=?$

38. Write in figures two hundred and six quadrillion thirty-four billion four hundred and forty-four million twenty-four thousand and three, *and* seventeen million two hundred and eighty-two one-hundred-quintillionths.

39. $976\triangledown1423\div\triangledown000\,007=?$

40. $3\triangledown70\dot{2}\dot{3}\div2\triangledown\dot{3}4\dot{5}=?$

41. $\dfrac{(\overline{2\frac{7}{8}+\triangledown5625}-\overline{1\triangledown5+\frac{1}{16}})\div\frac{11}{8}}{\{(1\frac{8}{11}\times\frac{4}{9}\times297\times\frac{1}{101})\div\frac{11}{8}\}\div9472947}=?$

42. $\dfrac{\overline{4\frac{2}{7}+\frac{8}{9}}-\frac{7}{12}}{\frac{3}{4}\text{ of }\frac{8}{13}\text{ of }\frac{1}{6}\text{ of }\frac{5}{9}}=?$

43. $\dfrac{\frac{19}{6}}{(\overline{2\frac{1}{3}\times\triangledown5\text{ of }1\frac{5}{7}}+9\frac{17}{21}+\triangledown\dot{0}\dot{9})\div\frac{23}{231}}=?$

44. $\dfrac{3\frac{1}{7}}{\{11\frac{6}{17}\div(\frac{11}{51}\text{ of }\triangledown1\dot{6})\}+(\frac{7}{10}\text{ of }3\frac{1}{5})}=?$

45. $$\frac{(\triangledown 76\dot{3}276\dot{3}\times 11\times\frac{1}{8}\text{ of }\frac{101}{106}\times\frac{1}{2}\text{ of }\triangledown 2\text{ times }\triangledown\dot{3}\text{ of }\triangledown 25\text{ of }96)\div\triangledown\dot{2}}{(\frac{1}{4}\text{ of }\triangledown 67\dot{3}246\dot{7})\div\frac{1}{9}}=?$$

46. $$\frac{5\frac{5}{8}\div\frac{2}{8}}{(1\frac{1}{5}\text{ of }\frac{5}{9})\div 10\frac{1}{8}}\times\frac{3}{5}\text{ of }\frac{1\frac{1}{2}\text{ of }4\frac{1}{9}}{13\frac{7}{8}\text{ of }5\frac{1}{8}}=?$$

47. How many h. in $\triangledown 06412583\dot{2}$ of $\frac{3}{5}$ wk.?

48. $\frac{1}{2}\times\triangledown\dot{3}=?$

49. Distribute 880 dollars, 2 dimes, and 6 cents among 3 men, 3 women, and 10 children so, that the men shall have equal shares as between themselves; the women, as between themselves; and the children, as between themselves likewise. But give to each man just thrice as much as to a woman and to each woman just twice as much as to a child.

a. How much will a child get?
b. How much will a woman get?
c. How much will a man get?

50. $16\frac{7}{11}+19\frac{4}{5}+129\frac{6}{7}-23\frac{7}{8}=?$

51. $(93\frac{4}{11}-76\frac{17}{23})\div\frac{17}{258}=?$

52. $6032\triangledown 5$m $\div$ 19 = how many dm?

53. Find the l.c.m. of 7, 11, 63, and 30.

54. What is the l.c.m. of 77, 42, 27, 21, 33, and 14?

55. Multiply seventy-two Mg, three Hg, four g, and nine cg by $217\frac{1}{8}$, and the product thus obtained by $\triangledown 504\dot{2}$.

56. Add $\triangledown 8$, $\triangledown 7\dot{5}$, $\triangledown 91\dot{9}8\dot{3}$, and $\triangledown 0033\dot{7}\dot{1}$; and then divide their sum by $\triangledown 14$.

57. Find the G.C.M. of 2691, 11 817, and 9828.

58. Divide 338 066 330 by 2 551 444 and add $\triangledown\dot{9}\dot{1}$ to the quotient thus obtained.

59. What is the weight of the water contained in a rectangular cistern 2 m wide, 3 m long, and $4\triangledown$ m deep, the cistern being full and the water at its greatest density?

60. Reduce $11 to di.

61. What is the capacity in s of a bin 4 m × 2v4 dm × 1 Dm?

62. What are the prime factors of 7680?

63. What is the value of a pile of stone 1v1 Dm long, 15 dm wide, and 355 cm high @ $2 per s?

64. What is 5v4 321 987 T of wheat worth @ $v004 per Hg?

65. Add $72v51 to $663v99, multiply their sum by 4v84, and distribute the product equally and evenly among 11 persons. How much will each then get?

66. Find the G.C.M. of 21 389 and 18 078.

67. $\frac{7}{11}+\frac{4}{5}+1\frac{2}{7}+\frac{8}{33}+\frac{11}{14}+\frac{7}{10}+\frac{1}{2}=?$

68. Purchased 14 m of cotton @ 11 c. per m, 19 m of ribbon @ 37½ c. per m, 17¼ m of silk @ $3 per m, a parasol for $4v65, a bonnet for $11v5, 6v7 Dm of sheeting @ 23 c. per m, and 9 m of French merino @ $1 per dm. How much did I spend?

69. How much is the half of $(\frac{5}{6}\times\frac{3}{4})\div(7-2\frac{7}{8})$?

115. RATIO.

The quotient obtained in division is the (geometrical) ratio between the divisor and the dividend, these being of the same kind, i. e., having the same unit. (What is sometimes termed "arithmetical ratio" is better designated by the word "difference"). The divisor and the dividend are called the terms of the ratio and constitute together what is often termed a couplet, (which virtually amounts to but another name for a fraction).

If the dividend be divided into the divisor, the quotient will be their inverse ratio, or reciprocal.

Since $x=\frac{x}{1}$, and since the quotient obtained by dividing any fraction into 1 is that fraction inverted, it follows that the reciprocal of *any* number is 1 divided by that number and that the reciprocal of a fraction is simply that fraction inverted. Thus $9=\frac{9}{1}$, i. e., $9\div1$; $11=\frac{11}{1}$, i. e., 11 divided by 1. Hence the reciprocal of 9 is $\frac{1}{9}$; that of 11 is $\frac{1}{11}$, i. e., $1\div11$; that of $\frac{2}{7}$ is $\frac{7}{2}$, (i. e., $1\div\frac{2}{7}=\frac{7}{2}$); and so of *any* number.

If we multiply together corresponding terms of 2 or more (simple) ratios, we will get a compound ratio. Thus $\overset{3}{\cancel{9}}\div\overset{1}{\cancel{3}}=3$; $\overset{5}{\cancel{25}}\div\overset{1}{\cancel{5}}=5$; $\overset{15}{\cancel{225}}\div\overset{1}{\cancel{15}}=15$.

$3\times5\times15=225$. And

$1\times1\times1=1$.

a. Ratios are compounded by multiplying all the divisors together for a new divisor and all the dividends together for a new dividend. Since the "ratio" is but the quotient, it follows that the principles enunciated in Points 57 to 62 will apply here likewise. And since any number of ratios to be compounded together may be expressed simply as so many fractions and then compounded by the method laid down in Point 104, it also follows that,

β. When several ratios are to be compounded, we may, before multiplying the corresponding terms together, cancel any factor that is common to a divisor and to a dividend.

116.

Exercises.

Find

a. The direct;

b. The inverse ratio of,

1. 12 to 3;
2. 14 Mm to 14 dm;
3. $\frac{1}{24}$ to $\frac{1}{36}$;
4. 10 to 1;
5. 18 s to 4 Dl;
6. 525 to 525;
7. 48 to $\frac{1}{12}$;
8. $\frac{1}{4}$ to twelve;
9. 16 Ha to 4 ca;
10. 1 T to 1 g;
11. 28 to 7;
12. 7 to 11;
13. 3 to 27;
14. 9 to 3;
15. 27 to 9;
16. 43 to 5;
17. 72 to 18;
18. 5 to 25;
19. 25 to 5;
20. 49 to 7;
21. 84 to 7;
22. 187 to 11;
23. 38 to 152;
24. 23 to 199;
25. 199 to 23;
26. 147 to 21;
27. 15 l to 2 cl; and
28. 21 to 7.

117. α'. If the divisor in any given couplet is just *equal* to the dividend, the ratio is equal to 1 and is called a ratio of equality.

β'. If the divisor is less than the dividend, the ratio will be greater than 1 and so a ratio of greater inequality.

γ'. If the divisor is greater than the dividend, the ratio will be less than 1 and so a ratio of less inequality.

Thus,

a. The ratio of 6 to 6=1, a ratio of equality;

b. The ratio of 6 to 3=2, a ratio of greater inequality;

c. The ratio of 6 to 12=$\frac{1}{2}$, a ratio of less inequality.

118. Ratios are compared with one another by expressing them in the form of a fraction, reducing these to equivalent fractions having a common divisor, and then comparing the dividends; or, (and this will very often be found to be the more convenient way), by working out the divisions indicated and then comparing the resultant quotients. The dividend of a fraction can always be regarded as being a whole number practically speaking under the name indicated by its divisor.

119. Compare both ways the 3 following sets of ratios:

a. 3 to 4, 7 to 8, and 9 to 10;
b. 7 to 8, 2 to 3, 11 to 13, and 5 to 6;
c. 7 to 33, 11 to 49, and 21 to 106.

120.

PRINCIPLES.

α′. If the terms of 2 or more couplets having the same ratio be added together divisors to divisors and dividends respectively to dividends, the resulting couplet will also have the same ratio.

β′. If from the terms of any couplet the terms of another couplet having the same ratio be subtracted divisor from divisor and dividend from dividend respectively, the resulting couplet will again have the same ratio.

γ′. A ratio of greater inequality is diminished by adding the same number to both its terms, and vice versa; whilst one of less inequality will thus be increased, and vice versa.

121.

ILLUSTRATIONS.

α′. The ratio of 6 to 2 is 3;

β′. The ratio of 24 to 8 is also 3;

γ′. Hence the ratio of 6+24, (i. e., of 30) to 2+8, (i. e., to 10) is 3 too.

δ′. The ratio of 35 to 5 is 7;

ε′. The ratio of 14 to 2 is 7 likewise;

ϛ′. Hence the ratio of 35−14, (i. e., of 21) to 5−2, (i. e., to 3) is 7 also.

ζ′. The ratio of 8 to 48 is $\frac{1}{6}$;

η′. But the ratio of 8+12, (i. e., of 20) to 48+12, (i. e., to 60) is $\frac{1}{3}$;
θ′. Whilst the ratio of 8—4, (i. e., of 4) to 48—4, (i. e., to 44) is but $\frac{1}{11}$.
ι′. The ratio of 49 to 7 is 7;
ια′. But the ratio of 49+14, (i. e., of 63) to 7+14, i. e., to 21) is 3 only;
ιβ′. Whilst the ratio of 49—4, (i. e., of 45) to 7—4, (i. e., to 3) is 15.

122. PROPORTION.

Proportion is an equality of ratios.

Thus, 18 to 3=30 to 5, and therefore this constitutes a proportion.

The terms of the 2 couplets are called proportionals.

In proportion it is customary to write merely the 4 *ends* of the sign of equality, thus ::, and, instead of "to" or "is to" between the terms of a couplet to indicate ratio, the ÷ is written with the dash left out. The above proportion is therefore written, 18:3::30:5, (read 18 is to 3 as 30 is to 5).

Since proportion is an *equality* between 2 ratios, what is to one side of the sign of equality can never vary from what is on the other; and since each ratio has 2 terms, the proportion will needs have 4. One *ratio* may be greater or less than another. This is impossible in proportion, which is quite another thing.

The outer terms of a proportion are called the extremes; the intermediate terms, the means.

If 4 quantities be proportionals, the product of the extremes will be just equal to that of the means.

If 6:11::18:33, then $6\times33=11\times18$; for $6:11=\frac{6}{11}$, and $18:33=\frac{18}{33}$. But if 6:11::18:33, then their equivalents, $\frac{6}{11}$ and $\frac{18}{33}$ must also be equal; wherefore $\frac{6}{11}=\frac{18}{33}$. Now, since multiplying equals by the same number cannot destroy their equality, multiplying these 2 fractions by 11 will not interfere with their equality, (Point 58). This multiplication will give us $\frac{6\times11}{11}$ and $\frac{18\times11}{33}=\frac{66}{11}$ and $\frac{198}{33}$, making $\frac{66}{11}=\frac{198}{33}$. Wherefore $6=\frac{18\times11}{33}$, i. e., to $\frac{198}{33}$. Now multiplying each of *these* by 33, we get $6\times33=18\times11$. But 6 and 33 are the extremes and 18 and 11 are the means.

If the product of the extremes of a geometrical proportion equals that of the means, the 4 quantities are proportional, otherwise contra. That is to say, otherwise it is not a proportion at all in the present sense.

123. Herefrom we deduce the following principles:—

α′. If the product of the means be divided by one of the extremes, the quotient will be the other extreme;

β′. If the product of the extremes be divided by one of the means, the quotient will be the other mean;

Wherefore:

γ′. If any 3 terms of a proportion be given, the fourth may be found, thus:

$$\text{1st term} = \frac{\text{2d} \times \text{3d}}{\text{4th}};$$

$$\text{2d term} = \frac{\text{1st} \times \text{4th}}{\text{3d}};$$

$$\text{3d term} = \frac{\text{1st} \times \text{4th}}{\text{2d}};$$

$$\text{4th term} = \frac{\text{2d} \times \text{3d}}{\text{1st}}.$$

124. A geometrical proportion will necessarily have the following properties:—

α′. If, for instance, 21:6 :: 14:4; then,

β′. Alternately, 21:14 :: 6:4;

γ′. Inversely, 6:21 :: 4:14;

δ′. By comparison, 21+6:6 :: 14+4:4, or 27:6 :: 18:4;

ε′. By division, 21−6:6 :: 14−4:4, or 15:6 :: 10:4;

ς′. By conversion, 21:21+6 :: 14:14+4, or 21:27 :: 14:18; or

ζ′. Again by conversion, 21:21−6 :: 14:14−4, i. e., 21:15 :: 14:10.

125. Simple proportion is often called the *Rule of 3.*

Proportion is also called the *Golden Rule* from its extensive utility.

Simple Proportion is an equality between 2 simple ratios.

Wherefore,

a. If one of these be a ratio of less inequality, the other must also be a ratio of less inequality;

b. If the one be a ratio of greater inequality, the other, too, must be a ratio of greater inequality.

Compound Proportion is an equality between a *compound* ratio and a simple ratio. It is also known as the double rule of 3. It is but a shorter way in which, in and by a single statement and operation, to find an answer, which, by simple proportion, would require 2 or more statements and operations. In every compound proportion problem, therefore, there are 3 or more ratios, one of which is imperfect.

126. RULE OF 3.

α. Draw a perpendicular line;

β. Write the given terms on both sides of this line so as to have terms of the same denomination, i. e., having, (or being reducible to) the same unit, opposite one another, thus forming pairs of all the perfect ratios and leaving the one given term of the imperfect ratio without an opposite;

γ. Strike out all ratios of *equality;*

δ. Draw a horizontal line;

ε. Over this line place the said given term of the imperfect ratio as a factor of the dividend of the fraction you are about to create;

ζ. Considering the first of the pairs given in connection with this term, and, where, as in compound proportion, there are still other terms, ignoring all these other terms for the moment, ask yourself: "Do these 3, independently of all the *other* terms, call for an answer greater or less than the said given term of the imperfect ratio?" If they call for a greater, place the greater of the 2 terms composing the pair in question above the horizontal line and after the term already there with an × between them, and the other term of said pair place beneath the (said horizontal) line; otherwise contra.

η. Treat the next remaining couplet in the same manner, then the next, and then the next, and so on throughout, always placing an × between every 2 terms so placed in the fraction, whether above or below;

θ. Work out now the multiplications and the division indicated, cancelling first any and all factors common to both divisor and dividend, or, which is the same thing, to any one factor of the divisor and to some one factor of the dividend.

127. ILLUSTRATIVE EXAMPLES.

If 15 s of flour cost $1600, what will 13 Ds cost?

Operation.

$$\left.\begin{array}{l|c} \text{s } 15 & 130 \\ \$1600 & — \end{array}\right\}$$

NOTE.—*13 Ds = 130 s. It would be more* LOGICAL *perhaps to put a note of interrogation where the dash is, but "—" is more easily and conveniently distinguishable from digits than "?".*

$\frac{1600\times\cancel{1}\cancel{3}\cancel{0}^{26}}{\cancel{1}\cancel{5}^{3}}$, or $(1600\times26)\div3=13866\triangledown\dot{6}$, i. e., dollars.

EXPLANATION.

As the dollars has no opposite, it is the given term of the imperfect ratio and so of the same denomination as the answer sought, wherefore we place it above the horizontal line. Then we ask, "If 15 s cost $1600, will 130 s cost more (than this $1600) or less?" Evidently more. Wherefore we place the 130, which is the greater term of this pair or couplet, above the line, (as a factor of the dividend of the fraction forming), and the 15 beneath. Now, since 5 is a factor common to one of the factors of the divisor (of this fraction), and at the same time to one of the factors of the dividend thereof, we cancel (by) it, thus reducing the 15 to 3 and the 130 to 26. The 2 factors of the dividend are now found to be 1600 and 26, which, when multiplied together as indicated =41600. Dividing this by the 3 left in the divisor makes it $13866▿$\dot{6}$, which will be the answer sought.

If 18 men dig a trench 30 m long in 24 d. by working 8 h. a d., how many men will dig a trench 6 Dm long in 64 d., working 6 h. a d. and both trenches being 1▿5 m deep?

Operation.

$$\left.\begin{array}{lr|l} \text{m dp.} & 1\triangledown5 & 1\triangledown5 \\ \text{men} & 18 & — \\ \text{m lng.} & 30 & 6\ \text{Dm lng.} \\ \text{d.} & 24 & 64 \\ \text{h.} & 8 & 6 \end{array}\right\} \doteq \begin{array}{lr|l} & & \\ \text{men} & 18 & — \\ \text{m lng.} & 30 & 60 \\ \text{d.} & 24 & 64 \\ \text{h.} & 8 & 6; \end{array}$$

$$\frac{\overset{9}{\cancel{18}}\times\overset{2}{\cancel{60}}\times\overset{3}{\cancel{24}}\times\overset{1}{\cancel{8}}}{\underset{1}{\cancel{30}}\times\underset{{}^{1}\cancel{8}}{\cancel{64}}\times\underset{\cancel{2}}{\cancel{6}}}=\frac{18}{1},=18.$$

128. EXERCISES.

1. If a man can walk 250 Cm in 12 d., how many Cm can he walk in 60 d.?

2. How much will $\frac{1}{4}$ m of cloth come to, if $\frac{7}{8}$ m cost $\frac{5}{6}$ dollars?

3. If $\frac{1}{8}$ of $\frac{3}{5}$ of $3\frac{1}{8}$ m of broadcloth cost $\frac{2}{7}$ of $\frac{3}{11}$ of $4\frac{2}{3}$ dollars, what will $\frac{55}{32}$ m cost?

4. When Jennie Rafferty had traveled 68 d. at the rate of 19 Cm a day, Lizzie Doyle, who started from the same point and traveled 48 d., overtook her. How many Mm did Lizzie travel per d.?

5. If 34v6995058 T of beef were cut up into pieces of 1 Cg, 1 Hg, 1 Dg, and 1 dg respectively so as to have an equal number of each,

a. How many pieces would there be?
b. How many of each kind?

6. Suppose Boxer, my greyhound, makes 28 springs whilst a fox makes 23 of equal length, how many bounds must Boxer make in order to overtake the fox, the latter having a start of 50 leaps?

7. If the interest on 1 dollar for 1 y. is 7 c., what will be the interest on $565 for 3v6̇ y.?

8. If 180 bricks 2 dm long, 1 dm wide, and 6 cm thick will be required to build a wall 3 m long, 5 dm high, and 2 dm thick; how many bricks v1 m wide, 1v8 dm long, and 7 cm thick will be required to build a wall 9 m and 8 cm long, 1v8 m high, and 1 m thick?

9. If 7 Hl of wheat will make 100 Cg of flour, how many Dl of wheat will it take to make 15 Q of flour?

10. If the annual rent of 17 Ha, 3 a, and 90 ca of land be $275v; how much land can be rented at that rate for 3 y. for $375v?

11. If, by selling $5000 worth of dry goods, Peter Synnott gains $456v25, for what amount must he sell goods so as to gain $1000?

12. If 12 Hl of wheat are bought for $44v5, and sold for $33v55; what would be the amount lost on 2 s of wheat at the same rate?

13. If it costs 412½ di. to pave a walk 2 m wide and 3 Dm long, how many E. will it cost to pave similarly a walk 263 cm wide and 2 Hm long?

14. What sum of money will earn $300 interest in 8 mo., if $800 earn $70 in 1¼ y.?

15. What can be the cost of 36½ m of cloth 1¼ m wide, if 2½ m 1v4 m wide cost $3v423?

16. If 2 m of cloth 1▿3 m wide can be made from 2 Cg of wool, how many m of cloth 1 m wide can be made from 3▿9 Mg of wool?

17. Auguste Dietner has hay worth $18 per T, and William McGlenn has flour worth $10 a barrel. If Mr. Dietner ask $21 a T for his hay, what must Mr. McGlenn ask for his flour to make an equitable exchange?

18. If a bin 2▿5 m long, 1▿5 m wide, and 9 dm deep will hold a certain given quantity of barley, how deep must I make a bin that will be 1 Dm long and 1⅝ m wide and calculated to hold 8 times as much barley?

19. What will 6 Mg of coffee cost, if 5 Cg of sugar cost $1▿25, and 8 Cg of sugar are worth 25 Hg of coffee?

20. Alexandre McLean has a farm worth $487▿5 and Daniel Sweeney has one worth $431▿25; but in trading Mr. McLean values his farm at $600. What value should Mr. Sweeney now put upon his?

21. If 96 Hl of wheat cost $300, what will 15 Hl cost?

22. If 27 men can mow 24 Ha of grass in 2 d., how many a can 93 men mow in 3 d.?

23. If 11 baskets of peaches cost $9, what will 37 baskets cost?

24. If 25 s of wood cost $141, what will 21 s cost?

25. If a man receive $22 for 16 days' work, how many days should he work for $88▿875?

26. If 16 bags of potatoes are sold for $11, what will 159 bags fetch?

27. If a stick 7 m long casts a shadow 5 m long, what must be the height of a tree that will cast a shadow of 112 m at the same time and also on level ground?

28. If a stack of hay will feed 27 cows 99 d., how long will it feed 55 cows?

29. If 9 Hl of pease sow 5 Ha of land, how many l will be required to sow 48485 ca?

30. If 3 men put up 73 Dm of fence in 2 d., working 8 h. a day, how long will it take 4 men to put up 80▽3 Hm of fence working 8 h. a day?

31. If 176 Dl of maple sap will make 60 Cg of sugar, how many Mg of sugar will 1 s of sap make?

32. If it costs $20▽88 to weave 188 m of cloth, what will it cost to weave 470 m?

33. If $16 will pay for the carriage of 72 s of flour, for the carriage of how many l of flour will $111 pay?

34. If 11 men plow 66 Ha of land in a wk., how many ca would 3 men plow in 1 wk.?

35. If 4 s of flour will make 2500 loaves of bread, how many such loaves will 67 s make?

36. If 80 Hl of apples make 1 s of cider, how many s of cider will 13 Hl of apples make?

37. If 90 men can build a wall in 12 d., how many men could build it in 15 d.?

38. If 17 days' work pays for 2 Hl of flour, for how many Dl of flour will 289 days' work pay?

39. If a train travel 44 Cm per h., how far will it travel in 2 d.?

40. If 7 cows make 16 Cg of butter per wk., how much butter will 28 cows make per wk.?

41. If $\frac{2}{5}$ of a basket of peaches cost $\frac{3}{7}$ of a dollar, what will $\frac{8}{11}$ of a basket cost?

42. If $\frac{9}{16}$ Dl of wine is worth $\frac{4}{11}$ Cg of coffee, how much coffee will $\frac{11}{12}$ Hl of wine be worth?

43. If $\frac{3}{16}$ of a ship is worth $9750, what is $\frac{21}{26}$ of it worth?

44. If $7▽49 will pay for $\frac{7}{9}$ of a T of coal, what will 8$\frac{1}{8}$ T cost?

45. If 5$\frac{4}{5}$ m of broadcloth cost $28▽42, what will $\frac{4}{7}$ Dm come to?

46. If $\frac{18}{25}$ dollars will pay for ▽8 bags of apples, for what part of a bag, (i. e., for how many bags) will $\frac{1}{20}$ dollars pay?

47. If $100 in stock is worth $98v875, what will $$475\frac{11}{25}$ in stock be worth?

48. If $17\frac{3}{5}$ T of hay last in a certain stable 102 d., how many wk. will $11\frac{11}{17}$ T last?

49. If 22 s of wood will last as long as 5 T of coal for fuel, how much wood would last as long as $11\frac{7}{28}$ T of coal?

50. If 11 s, 1 ds, and 1 cs of grain cost $75, what will 716 Dl and 3 Hl cost?

51. What will 37 ca and 4 a of painting cost, if 9 ca and 2 a cost $100?

52. How much will 7 Cg of silver come to @ $1 per Dg?

53. If 10 m of ribbon cost $3v5, what will 3v8 m cost?

54. If 1 Hg and 2 Dg of silver cost $18v8, what will 1 Cg, 2 g, and 3 dg be worth?

55. If 6 Cg of aluminium be worth $500, what will be the value of 2 Cg, 3 Dg, 1 g, and 9 dg of aluminium?

56. If a man can pump 6 s of water in 2 h., 16 m., and 33 s.; in what time will he pump 4 s, 1 Ds, and 1v Dl?

57. What will 146 m of velvet cost, if 32v12 m cost $22?

58. If 48v g of silver cost $2, what will 10 Mg and 8 dg cost?

59. In the copy of a work of 327 pages a certain passage commences at the end of the 156th page. On what page might it be expected to begin in a copy thereof of 400 pages?

60. If the rent of 19v Ha of land be $500, what will be the rent of 15v2 Ha?

61. If $$6\frac{2}{3}$ will pay for $8\frac{2}{3}$ Cg of prunes, how many Dg of prunes can be bought for $$6\frac{7}{8}$?

62. A T of coal yields about 1 Hs of gas; a street lamp consumes about 1 Hl and an argand burner, say 5 Dl of gas in an hour. How many T of coal would be needed to keep 17493 street lamps and 192 724 argand burners lighted for $4\frac{1}{2}$ full days (of 24 h. each) at that rate?

63. If 5 compositors in 16 d. of 11 h. can set up 25 sheets of 24 pages each, 44 lines in a page, and 40 letters in a line; in how many d. of 10 h. may 9 compositors be expected to set up a volume to be printed in the same type and consisting of 36 sheets, having 16 pages to the sheet, 50 lines to a page, and 45 letters to a line?

64. If 24 men can saw 190 s of wood in 6 d. of 9 h. each, how many s can 8 men saw in 36 d. of 12 h. each?

65. If 248 men in $5\frac{1}{2}$ d. of 11 h. each dig a trench 235 m long, 3 m wide, and 2v2 m deep; in how many d. of 9 h. each will 124 men dig a trench 337v5 m long, 5 m wide, and $9\frac{2}{5}$ dm deep?

66. If 120 Hl of oats last 14 horses 175 d., how many wk. will 9 Cl of oats last 6 horses?

67. If a wall 9 m high was built in 16 d. by 63 men, how many men would build a wall 1v1 Dm high in 8 d.?

68. If 1 Cg of thread make 5 m of linen of 9 dm width, how many Mg of thread would be required to make a piece of linen 5 Dm long and 1v001 m wide?

69. If 3 Cg of worsted will make 2 Dm of stuff 1v4 m broad, how many Dg of thread will make a piece 1 Hm, long and 16 dm wide?

70. If it takes $1 one y. to draw 8 c. interest, how many E. will have to be loaned out so that they will draw in five y. and 6 mo. $100 at that rate?

71. If 12 horses in 5 d. draw 44 T of stone, how many horses would draw 132 T over the same road in 18 d.?

72. If 4 men earn $72 in 8 d., how much will 14 men earn in 6 wk.?

73. If 2 mechanics can earn $244 in 5 wk., how much would 3 mechanics earn in 6 wk.?

74. If 7 shoemakers in 4 wk., working 5 d. in the week and 7 h. per day, make 105 pairs of shoes; how many pairs of shoes would 15 equally able and industrious shoemakers make in 3 wk., working 5v6 d. per week and 6 h. per day?

75. A wall is to be built 9v3 m long, and 2v3 m of its length are built by 12 men in 6 d., working 9v2 h. a day. How many men must be employed to now finish the wall in 4 d., working 12 h. per day?

76. If Robert Haire travel 200 Cm in 3 d. of 12 h. each, in how many days of 9 h. each will he go 60 Mm?

77. If the price of 30 Hg of bread, when flour is 6 c. per Cg, is 20 c.; what must be paid for 18 Cg of bread when flour is 1 c. per Hg?

78. If I must pay $7 for the use of $95 for 1 y., what will the use of $361 for 5¼ y. cost me?

79. If $100 will cost David Dablon $8 interest in one year, what will $564v cost him in interest, if he use them for 4 y. and 4 mo.?

80. If a pasture field of 7 Ha will feed 6 horses for 4 mo., how many a will it take to feed 12 horses for 9 months?

81. If 25 persons consume 21 Ml of wheat in one year, how many Cl of wheat will 139 persons consume in 7 y.?

82. If 32 men can build a wall 12 m long, 2 m high, and 1v4 m wide in 4 d.; in what time will 48 men build a wall 1 Cm long, 1 Dm high, and 1 m thick?

83. If a regiment of 671 soldiers consume 222 Hl of wheat in 1 common year, how many Cl of wheat will an army of 22 440 soldiers consume in 1 leap year?

84. If 12 tailors in 20 d. can finish 13 suits of clothes, how many tailors in 19 d. can finish 494 suits?

85. If 17 head of cattle consume 2 Ha and 25 a of pasturage in 30 d., how many ca would be consumed by a herd of 40 head in 51 d.?

86. If 4 T of coal cost $24, what will 120 Q cost?

87. If 4 horses eat 5 Hl of oats in 6 d., how many l of oats will 20 horses eat in 7 d.?

88. If 24 s of wood cost $50, what will 40 s cost?

89. If 11 Cg of sugar cost $▿81, what will 24 Cg cost?

90. If 6 Hl of wheat will make 4 barrels of flour, how many barrels of equal size will 7 Cl of flour make?

91. If 8 men earn $32 in 2 d., how many di. will 15 men earn in 6 d.?

92. If 5 horses consume 10 T of hay in 8 mo., how many horses will consume 1 000 000 Mg in ⅔ y.?

93. If 8 m of cloth cost $6, how many m can be bought for $76▿4?

94. How many men will be required to build 16 Dm of wall whilst 5 men build 500 dm?

95. If 5 sheep can be bought for $20▿, how many sheep can be bought at the same price for 40 E.?

96. When 10 Mg of flour cost $11▿25, what will 4▿76 Q cost?

97. If a railway train run 5 Mm in 50 m., in what time will it run 40 200 Hm?

98. How many Dl of peaches can be purchased for $454▿4, if 3 Hl cost $10▿?

99. If a horse travel 1▿9 Mm in 1 h. and 30 m., how many Dm will he travel at that rate in 15 h.?

100. How many d. will 12 men require to do a piece of work that 95 men can do in 7½ d.?

101. If 15 a of land cost $60, what will 20 Ha of it cost?

102. At the rate of 71 m for $222, how many Dm of broadcloth can be bought for $28▿?

103. If ⅞ barrels of flour cost $14, what will ⅚ of ▿9 barrels cost?

104. What will 87▿2 m of cloth cost, if 18 dm cost $1¼?

105. Bought coal @ $4▿48 per T and sold it @ 73 c. per Q. What was the gain per Cg?

106. What will be the cost of a pile of stone 22 m long, 13 dm wide, and ▿14 Dm high; if a pile 391▿3 dm long, ▿11 Dm wide, and ▿02 Hm high will cost $21▿5?

107. If Charles Lennan's business yields him $700 net profit in 1 y. and 8 mo., in what time would it yield $1050?

108. If 18 men build 126 Dm of wall in 40 d., working 10 h. a day, how many m of wall will 6 men build in 111 d., working 11 h. a day?

109. If 20 horses consume 36 T of hay in 9 mo., how many T will a herd of 12 horses consume in 19 mo.?

110. If $320 will pay the board of 4 persons for 8 wk., for how many wk. will $800 pay the board for 15 persons?

111. If a man can walk 30 Mm in 6 d., walking 8 h. a day; how far could he walk in 18 d., walking 6 h. a day?

112. If 6 laborers can dig a ditch 33 m long in 10 d., how long would 20 laborers need to dig one 1v9 Hm long?

113. If 8 men earn $320 in 8 d., how many cents will 12 men earn in 4 d.?

114. How many days will 21 men require to dig a ditch 2v73 Dm long, 1 m wide, and 1⅓ m deep; if 7 men can dig one 2 Dm long, 12 dm wide, and 2v1 m deep in 12 d.?

115. If 4 horses consume 17 Hl of oats in 12 d., how many Cl will 20 horses consume in 8 wk.?

116. If 20 reams of paper are required to print 300 copies of a book containing 230 pages having 40 lines on a page, how many reams will be required to print 3000 copies of 400 pages having 35 lines on a page?

117. If 10 men can cut 146 s of wood in 18 d., working 10 h. a day, how many s can 40 men cut in 24 d., working 9 h. a day?

118. A contractor employs 45 men to do a given job of work in 3 mo. What additional number of men would he have had to employ in order to be able to do it in 2½ mo.?

119. If a vat 5▿1 m long, 2 m wide, and 2▿2 m deep holds 384 barrels of water; how many barrels will a vat 5▿81 m long, 3 m wide, and 3▿4 m deep hold?

120. What is the weight of a block of granite $2\frac{1}{2}$ m long, $1\frac{1}{4}$ m wide, and 3 dm thick, if a block 3 m long, $1\frac{1}{3}$ m wide, and 4 dm thick weigh 2▿4 T?

121. If it cost $15 to carry 20 T and 5 Q, what will it cost to carry 400 T the same distance?

122. If it take 13 500 bricks to build a wall 58 m long, 6 m high, and 4 dm thick; how many bricks will be needed to build a wall 87 m long, 8 m high, and 5 dm thick?

123. What will 1▿5 l of molasses cost, if 1 Cl costs $$23\frac{1}{4}$?

124. At $6\frac{1}{4}$ c. for $1\frac{1}{8}$ m of cotton cloth, how many m can be bought for $$52\frac{1}{10}$?

125. If $750 gain $155 in 1 y. and 6 mo., what sum will gain $$201\frac{1}{2}$ in 4 y. and 6 mo.?

126. In what time can 60 men do a piece of work that 15 men can do in 21 d.?

127. If the use of $300 for 1 y. is worth $30, how long, at that rate, may $210▿25 be retained in use to be worth $42▿891?

128. How many men will be required to dig a cellar 16 m long, 1▿2 Dm wide, and 31 dm deep in 12 d. of 8 h. each; if 6 men can dig one 7▿2 m long, ▿5 Dm wide, and 3▿1 m deep in 3 d. of 10 h. each?

129. If 6 men, in 4 mo., working 25 d. per mo. and 12 h. a d., can set the type for 24 books of 300 pages each, 40 lines to the page, 12 words to the line, and an average of 6 letters to the word; in how many y., working 24 d. per mo. and 10 h. a d., can 8 men and 4 boys set the type for 10 books of 240 pages each, 50 lines to the page, 16 words in a line, and 8 letters on an average to the word, a boy doing just half as much as a man?

129. CONJOINED PROPORTION.

Conjoined Proportion is a sort of compound proportion in which the ratio of one of the terms to its corresponding term is made to depend upon equivalences among the intermediate terms of the proportion.

It is sometimes styled the "chain rule."

130. ILLUSTRATION.

Suppose 7 m of velvet in Toronto cost as much as 9 m in Montréal, and 16 m in Montréal cost as much as 24 m in Paris; how many m in Toronto will cost as much as 54 m in Paris?

Operation.

$$\begin{array}{lr|l} \text{M.} & 9 & 16 \\ \text{P.} & 24 & 54 \\ \text{T.} & 7 & — \end{array}$$

$$\frac{7\times\overset{4}{\cancel{1}\cancel{6}}\times\overset{\cancel{6}_1}{\cancel{5}\cancel{4}}}{{}_1\cancel{9}\times\cancel{2}\cancel{4}\ \cancel{4}_1}=\frac{28}{1},=28.$$

The same result will more readily and conveniently be reached in this kind of examples by stating and working them as follows, To Wit:

$$\begin{array}{lr|rl} \text{T.} & 7 & {}^1\cancel{9} & \text{M.} \\ \text{M.} & {}^4\cancel{1}\cancel{6} & \cancel{4}\ \cancel{2}\cancel{4}^1 & \text{P.} \\ \text{P.} & {}^1\cancel{6}\ \cancel{5}\cancel{4} & ? & \text{T.} \end{array}$$

Now, $1\times1=1;$
$1\times4\times7=28;$
$28\div1=28.$

Explanation.

Having placed 7 T., (Toronto) above the horizontal line as being the term minus an opposite, and so the given term of the imperfect ratio, we ask: "If 9 M., (Montréal) will pay for 7 T., will 16 M. pay for more or less?" Evidently for more. So we put the 16 above the line as one of the factors of the dividend and the 9 beneath as a factor of the divisor in the fraction thus being created. Then, proceeding to the next couplet, we enquire, "Will 54 P., (Paris) pay for more or for less than 24 P.?" More.

Wherefore we place the 54 above the horizontal line and the 24 below it.

Nevertheless in working conjoined proportion proceed rather as follows:

131. *α.* Write the equivalent terms, as they occur, right and left of a perpendicular line, taking care that terms of the same name, or that are reducible *to* the same name shall always be on opposite sides;

β. Reduce each pair of terms then to the same name if they be not already so;

γ. Cancel all factors occurring with equal frequency on both sides of the line;

δ. Multiply together,

a. All the (thus reduced and other) terms on the side of the line on which is found the odd term for a dividend,

b. All those on the other side for a divisor;

ε. Divide then accordingly.

132. EXERCISES.

1. If 25 sheep eat as much as 19 goats, and 33 goats as much as 10 cows, and 38 cows as much as 22 horses; how many horses will eat as much as a flock of 60 sheep?

2. If 19 Cg of tea in Guelph cost as much as 2 Mg in Hamilton, and 7 Cg in Hamilton as much as 6v5 Cg in Crieff, and 6 Cg in Crieff as much as 9 Cg in Quebec, and 3 Mg in Quebec as much as 29¾ Cg in Boston, and 85 Hg in Boston as much as 5⅓ Cg in London, and 1 Mg in London as much as 5v7 Mg in Hong Kong; how many Dg of tea in Hong Kong will be worth 1 Q of tea in Guelph?

3. If 17 s of wood are in value equivalent to 2 Q of tea, and 44 Cg of tea to 23 Hl of flour, and 1v9 Cl of flour to 34 days' work, and 92 days' work to 57 Dl of grapes, and

310 l of grapes to $24, and $12 to 2 T of coal; how many T of coal may be purchased for 62 s of wood?

4. If 12 T of tea are worth 29 T of sugar, and 17 Q of sugar pay for 10∇2 s of wheat, and 27 Hl of wheat pays for 4 T of coal, and 58 T of coal purchase 15 cows, and 29 cows cost $1160; how many dollars will buy 222∇2 Mg of tea?

5. If 39 Dl of barley pay for 7 Hl of potatoes, and 19∇6 Cl of potatoes pay for 2∇9 Ml of oats, and 115 l of oats pay for 52 l of wheat, and 145 dl of wheat pay for 36 l of pease, and 6 s of pease pay for 5½ Q of rye, and 7½ dg of rye pay for 11½ ml of clover seed; for how many cl of clover seed will 36 cl of barley pay?

6. If 48 baskets of grapes pay for 9 turkeys, and 17 turkeys for 7 days' work, and 4¼ days' work for 200 loaves of bread, and 8½ loaves of bread cost as much as 2 Cg of veal, and 1 Hg of veal is worth 2 c., and $7∇92 pay for 8∇1 Mg of sugar; how many baskets of grapes will 21 Dg of sugar buy?

7. Suppose Peter Brick can do as much work in 7 d. as Bernard Kaiser can do in 11 d., and Bernard can do as much in 5 d. as Thomas Abélard can do in 8 d., and Thomas can do as much in 15 d. as John Carroll can do in 21 d., and John can do as much in 11 d. as Johnny Hughes can do in 5 d.; in how many d. would Peter Brick do as much work as Johnny would do in 42 d.?

8. If 6 barrels of flour pay for 48 s of wood, and 6 s of wood will pay for 88 Cg of beef, and 5 Mg of beef cost $3∇1, and $77 will pay for 9 sheep, and 8 sheep are worth as much as 5 T of coal; how many barrels of flour of equal size may be purchased for 93 Q of coal?

9. If 16 sheep eat as much grass as 6 cows, and 25 cows require 100 Ha of pasture, and 3000 a of pasture supply 5 horses, and 10 horses eat as much as 23 goats; how many sheep will eat as much as 69 goats?

10. If whilst Margaret Anne Black said 10 words Alathea McCurry could say 11, and whilst Alathea said 55 words Mary Stievens could manage to say but 48 words, and whilst Mary would be saying 84 Grace McDonald could say only 81, and whilst Grace said 476 words Anne Foster could say 600, and whilst Anne spoke 99 words Maria Campbell could utter but 85, and Julia Anne Flood required as much time in which to say 15 words as Maria needed to say 16 but could say 102 words whilst Sara Jane Tobin could say 98, and Sara Jane could say but 65 words whilst Marie Shannon would be saying 99, and whilst Marie said 891 words Nellie Badeau could utter but 714, and whilst Nellie would be saying 75 words Barbe Schneucker could successfully pronounce 80, and whilst Barbe was saying 77 words Marguerite Barnet could say but 75, and whilst Marguerite would be saying 54 words Augusta Voisin could say 55, and whilst Augusta said 21 words Eloise Abélard said 22, and whilst Eloise said 350 words Jennet McCurry could say but 325, and whilst Jennet would be saying 11 words Irmine Howard could say but 9, and if Sophie Normandin could say 30 words whilst Irmine uttered 27 but could say only 100 whilst Kettie Carroll would be saying 110 words, then whilst Kettie would be saying 330 words Bride Conway could say 360; how many words would Margaret Anne Black be able to say whilst Bride would be rattling off 2000?

11. How many days are there from March 12th to the 17th of the following February, both days included?

12. From $76\triangledown23\dot{4}7\dot{8}$ take $19\triangledown134\dot{2}29\dot{1}$.

13. If nitric acid is $1\frac{1}{5}$ times the weight of water, what will $\triangledown79$ l of nitric acid weigh?

14. Divide 63 m, 3 dm, 2 cm, and 5 mm of riband equally among 17 girls. How much will each get?

15. What is the value of 9▿13625 a of land @ 64 c. per ca?

16. Multiply $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{8}$ of 20 by $\triangledown5\times\triangledown\dot{6}\times(\frac{7}{8}\div\frac{1}{4})$.

17. Of the ratios 6 to 7, 17 to 8, 23 to 11, and 88 to 176 point out,

a. Which is the greatest;
b. Which is the least;
c. Which, if any of them, are ratios of greater inequality;
d. Which, if any, are ratios of less inequality;
e. And what the direct ratio compounded of these is.

18. From one-half of two-thirds of eighteen twenty-ninths subtract one-eighth of two-thirds of five-sevenths.

19. What is 79 Cg of tea worth @ $▿001 per g?

20. If 3 men in 12▿5 d., working 12 h. a day, can cradle a field of wheat containing 81 Ha; in how many d. can 4 men, working 10 h. a day, cradle 1414▿2 a?

21. $(\frac{4}{5} \text{ of } \frac{9}{11}\times\triangledown02\times\triangledown\dot{4}5\dot{6})\div(\frac{16}{17} \text{ of } \frac{2}{3} \text{ of } \frac{1}{8} \text{ of } 51)=?$

22. A certain number is divided by 5; the quotient, by $\frac{7}{2}$, this quotient, by $\frac{13}{5}$; and then this last quotient, by $\frac{4}{7}$. The final result now being 2, what was the original number?

23. If 45 Q of flour in Tacoma are worth 155 m of cloth in San Francisco, and 8 Dm of cloth in San Francisco be worth 8 bales of cotton in Charleston, and 31 bales of cotton in Charleston be worth $4\frac{1}{2}$ T of sugar in New Orléans; how many Q of flour in Tacoma will be worth 1000 T of sugar in New Orléans?

24. $(73\triangledown47\times\triangledown0063)\div7\triangledown2\dot{3}=?$

25. Reduce 2 Ha, 7 a, and 40 ca to the decimal of 17 Ha.

26. $(\triangledown73 \text{ of } 11 \text{ Hm})-(\frac{3}{7} \text{ of } \frac{4}{5} \text{ of } \frac{1}{8} \text{ of } 71 \text{ m})=?$

27. Find the l.c.m. of 44, 275. 18, 190, 209, and 225.

28. If 60 men in 6 wk. of 5 working days each, working

10 h. a d., build an embankment 12 Hm long, 5 m wide, and 31 dm high; how many men will make a similar embankment 3▿1 Cm long, 6 m wide, and $2\frac{1}{5}$ m high in 30 d. of 6 h. each?

29. $42\triangledown 7\times 9\triangledown 7\dot{1}2\dot{3}=?$

30. If $\frac{2}{7}$ of $\frac{3}{8}$ of $3\frac{1}{2}$ Cg of coffee cost $\frac{9}{7}$ of $\frac{3}{5}$ of $\frac{20}{27}$ of $\frac{1}{2}$ of $1, what will $\frac{3}{8}$ of ▿7 of ▿6 of $\frac{21}{50}$ of 90 Mg cost?

31. If $4529 be divided among 7 men, 2 women, and 11 children so, that each child will have $\frac{2}{7}$ times as much as a woman, and each woman $\frac{3}{11}$ times as much as a man, the men sharing equally as among themselves; how much will,

a. A man get?
b. A woman get?
c. A child get?

32. What is the reciprocal (ratio) of $\frac{2}{7}$ to $\frac{11}{3}$?

33. What is the (direct) ratio of 93 to 17?

34. What is the inverted ratio of $\frac{2}{5}$ to $\frac{7}{8}$?

35. Add $\frac{4}{9}$ of $6\frac{1}{2}$ Dm, $\frac{3}{5}$ of $\frac{4}{7}$ of $8\frac{3}{4}$ m, and $\frac{2}{7}$ of $\frac{3}{11}$ of $7\frac{7}{10}$ dm.

36. What is the ratio compounded of 23 to 7, 4 to 11, 6 to 5, 13 to $11\frac{1}{2}$, and $38\frac{1}{2}$ to 3?

37. If 1 l will hold 2000 grains of barley and each grain is 6 mm long, how far would the barley grains in 9▿25526 Hl reach, set end to end?

38. Reduce $\frac{4158}{10395}$ to its lowest terms.

39. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{2}{7}$ together and then divide their sum by $\frac{4}{5}$.

40. Suppose that 58 men by working 11 h. each day can dig, in 54 d., 24 cellars which are each 1▿1 Dm long, 7 m wide, and 29 dm deep; how many men would be required to dig, in 27 d., 18 cellars each to be ▿1 Hm long, 9 m wide, and 350 cm deep, if they work but 6 h. a day?

41. $(4\triangledown\dot{3}\dot{1}+\triangledown 41\dot{3})\times(7\triangledown 2-3\triangledown\dot{8})\times(35\triangledown\dot{4}\div\triangledown\dot{7})=?$

133. PRACTICE.

When one of the three terms given in an example in proportion is 1, the work may be abridged, since multiplying or dividing by 1 works no change and is therefore but a superfluous operation, which had better be omitted altogether.

And this is what is done in what is called "Practice."

134. ILLUSTRATIVE EXAMPLES.

Find the price of 2783 m of silk @ $3v375 per m.

Operation.

m 1	2783
$3v375	—

$$\frac{3v375\times2783}{1}=\$9392v625.$$

What is 972 Dg of gold worth @ $1v15 per g?

Operation.

$\$1v15\times10=\$11v5$, price of 1 Dg;

$(972\times11\frac{1}{2})\div1=11v5\times972,=11178$, i. e., dollars.

135. EXERCISES.

1. Find the price of 729 days' work @ $$5\frac{1}{4}$ a day.

2. What will 624 Hl cost @ $2v8 a Hl?

3. What will be the price of 96 Ha, 49 a, and 30 ca of land @ $111v11 a Ha?

4. What will $964\frac{11}{15}$ ca of plastering cost @ $22\frac{1}{2}$ c. per ca?

136. It will sometimes happen, as in the last example above, that the 2 given numbers other than the 1 cannot well be multiplied together without first being reduced to their equivalent fractions. In such an emergency it will frequently prove to be easier and simpler to dissect one of these numbers or terms into parts, multiply the other by these parts successively, and then add the several products so obtained together. This is done by using aliquot parts. But take ever the easiest way.

An aliquot part of a number is such a part of it as will be indicated by a fraction having for its divisor the quotient obtained when the number is divided by one of its measures and for its dividend a 1.

137. Thus:

50 c. $=\frac{1}{2}$ of \$1;
$33\frac{1}{3}$ " $=\frac{1}{3}$. " ";
25 " $=\frac{1}{4}$ " ";
20 " $=\frac{1}{5}$ " ";
$16\frac{2}{3}$ " $=\frac{1}{6}$ " ";
$12\frac{1}{2}$ " $=\frac{1}{8}$ " ";
10 " $=\frac{1}{10}$ " ";
$8\frac{1}{3}$ " $=\frac{1}{12}$ " ";
$6\frac{1}{4}$ " $=\frac{1}{16}$ " ";
5 " $=\frac{1}{20}$ " ";
4 " $=\frac{1}{25}$ " ";
2 " $=\frac{1}{50}$ " " .

6 mo. $=\frac{1}{2}$ of 1 y.;
4 " $=\frac{1}{3}$ " " ";
3 " $=\frac{1}{4}$ " " ";
$2\frac{2}{5}$ " $=\frac{1}{5}$ " " ";
2 " $=\frac{1}{6}$ " " ";
$1\frac{1}{2}$ " $=\frac{1}{8}$ " " ";
$1\frac{1}{5}$ " $=\frac{1}{10}$ " " ";
1 " $=\frac{1}{12}$ " " " .

15 d. $=\frac{1}{2}$ of 1 mo.;
10 " $=\frac{1}{3}$ " " " ;
6 " $=\frac{1}{5}$ " " " ;
5 " $=\frac{1}{6}$ " " " ;
3 " $=\frac{1}{10}$ " " " ;
2 " $=\frac{1}{15}$ " " " .
This is counting even 30 d. to a month.

$\frac{1}{10}$ is the smallest *convenient* aliquot part of *anything*. Aliquot parts *of* aliquot parts, too, can sometimes be called into requisition to advantage.

138. By means of this aliquot part method the last above example would be worked as follows, To Wit:

$\frac{11}{15}$=v73̇, therefore $964\frac{11}{15}$=964v73̇;
20 c. is the $\frac{1}{5}$ of a dollar;
$2\frac{1}{2}$ c. is the $\frac{1}{8}$ of 20 c.

Now, if the 967v73̇ ca cost just one dollar per ca, the cost, (answer) would obviously be just \$964v73̇. But they cost 20 c. plus $2\frac{1}{2}$ c. per ca, wherefore we must find the $\frac{1}{5}$ of 964v73̇ and the $\frac{1}{8}$ of that $\frac{1}{5}$ and add them together; then the answer will be dollars. The $\frac{1}{5}$ of a number is found either by dividing it by 5 or by multiplying it by v2, (Point 108). In either case we will find that the $\frac{1}{5}$ of 964v73̇ is 192v946̇, of which the $\frac{1}{8}$ is found to be 24v1183̇.

139. *Operation.*

$$\begin{array}{r|c|l}
20\text{ c.}=\frac{1}{5} & 5 & 964\triangledown7\dot{3} \\ \cline{3-3}
2\frac{1}{2}\text{ c.}=\frac{1}{8} & 8 & \left.\begin{array}{r}192\triangledown94\dot{6}\\ 24\triangledown118\dot{3}\end{array}\right\}=\left\{\begin{array}{r}192\triangledown946\dot{6}\\ 24\triangledown118\dot{3}\end{array}\right\}=\$217\triangledown065.
\end{array}$$

140. EXERCISES.

1. Required the value of 92 647⅕ Cg of tea @ 69 c. per Cg.
2. What is the cost of 94 937 pails @ 3 di. each?
3. What are 95 972 paste-board boxes worth @ 7▿5 c. each?
4. What is the cost of 62 Ha of land @ $2▿2 per a?
5. Find the price of 2311 Mg of coffee @ 1 c. per Dg.
6. What is the price of 2103 s of wood @ $2▿1 per s?
7. What is the price of 211▿7 Hg of rice @ $95▿3 a T?
8. What will 7506 pairs of shoes come to @ $2▿11 a pair?
9. What will be the value of 1217 m of cotton @ $7½ per Hm?
10. What will be the cost of 2 096 Dg of gold @ $1▿12 per g?
11. Required the value of 1 Cg, 23 Hg, and 1 dg of silver @ 1 di. per g.
12. What is the cost of 98 m, 3 dm, and 1 cm of cloth @ $75 per Dm?
13. What will the rent for 34▿4315 Ha of land be @ $▿33 per a?
14. What would 1▿5 Hg of mercury come to @ $1▿5 per g?
15. Find the value of 4▿23 m of satin @ $5▿65 per m.
16. What will 321▿14 Ha of land be worth @ $23 per Ha?
17. Find the price of 13 l of wine @ $▿09 per l.
18. How much will 7▿24 Cl of apples fetch @ $3▿2 per Hl?
19. What will 721 Cg of wheat cost @ 3 c. per Cg?
20. What will 4514½ Dm of fencing cost @ $15▿ per Hm?
21. Divide 428▿159 by ▿0061.

22. What will 19▿193 275 T of hops cost @ $192 per T?

23. 73▿7̇27̇ + 11▿342̇ + 16▿713 + 19▿0̇54̇ + 713▿31 = ?

24. If 2 a of land cost $5▿5, what will 25 Ha, 3 a, and 94 ca cost?

25. What is the G.C.M. of 144, 485, and 63?

26. If 343 Dl of oats are worth 17 Hl of potatoes, and 9 Dl of potatoes are worth 1 Hl of flour, and 1▿3 Hl of flour is worth $6▿3, and 38 c. pay for 7 Cg of bread; how many oats will be worth 62▿05 Mg of bread?

27. If $157 will buy one horse, how many horses at this rate and price can you buy for $37 523?

28. What is the l.c.m. of the first or lowest 10 even numbers?

29. From 97▿91 3̇12̇ take 18▿123 4̇56̇.

30. What would be the cost of painting a ceiling 6▿5 m long and 514▿ cm wide @ 3 di. per ca?

31. Divide 916▿8 by 100 and then the quotient by 20.

32. 1001 + (91▿17 − 34) + (3▿762̇ ÷ 8▿8̇) + (763 × 3▿4̇14̇) = ?

141. PERCENTAGE.

Per cent, or rather *percentum* means in the hundred. The expression is used to indicate so much or so many in the 100. Thus 5 per cent, (written 5%) of a given number of dollars, litre, stère, mètre, gramme, or days, for instance, would mean $5 every hundred dollars or at that rate, 5 l out of every 100 l, 5 s out of every 100 s, 5 m for every 100 m, 5 g for or in every 100 g, or 5 d. out of every 100 d. as the case might happen to be. The rate per unit will, of course, be one one-hundredth part of the rate per cent, inasmuch as there are 100 units or ones in 100. It is therefore found by dividing the rate per cent by 100, i. e., by moving the delta 2 places leftward. Thus for example,

$1\% = \frac{1}{100}$ or ∇01 per unit;
$2\% = \frac{2}{100}$ or ∇02 per unit;
$9\% = \frac{9}{100}$ or ∇09 per unit;
$10\% = \frac{10}{100}$ or $\frac{1}{10}$ or ∇1 per unit;
$39\% = \frac{39}{100}$ or ∇39 per unit;
$125\% = \frac{125}{100}, = 125 \div 100$ or 1∇25, i. e., $1\frac{1}{4}$ per unit;
$\frac{1}{2}$ of $1\% = \frac{\frac{1}{2}}{100}$ or ∇005 per unit;
$\frac{3}{4}$ of $1\% = \frac{\frac{3}{4}}{100}$ or ∇0075 per unit;
$6\frac{1}{8}\% = \frac{6\frac{1}{8}}{100}$, i. e., 6∇125 $\div$ 100 or ∇06125 per unit.

143. EXERCISES.

1. What rate per unit is equivalent to 11∇3%?
2. What rate per unit is equivalent to $124\frac{1}{4}\%$?
3. What rate per unit is equivalent to $\frac{1}{2}$ of 1%?
4. At $23\frac{5}{8}\%$ how much is it per 1?
5. At $19\frac{1}{3}\%$ how much will it be per 1?

144. To find the percentage at any required rate on any given number,

Multiply the given number by that rate per unit.

145. ILLUSTRATIVE EXAMPLES.

What is 7% of 673▿93 dollars?

Operation.

7÷100=▿07;
673▿93×▿07=$47▿1751, answer.

What is 4½% of 3780?

Operation.

4½=4▿5, and 4▿5÷100=▿045 rate per unit.
3780×▿045=170▿1, answer.

146. EXERCISES.

1. What is 6½% of $2934?
2. What is 47¾% of 7893 l.?
3. What is 5% of $742▿1?
4. What is 11% of $1000?
5. What is 10% of $734▿19?
6. A merchant imports 2740 boxes of oranges and finds, upon their arrival, that 20% of the whole lot are decayed. To how many boxes was his loss equivalent?
7. Kate Barclay purchased a foundry for $7490, agreeing to pay 10% down, 17% at the end of the first year thereafter, 27% at the end of the 2d year, and 46% at the end of the 3d year. Required the amounts respectively,

a. Of the first payment;
b. Of the second payment;
c. Of the third payment;
d. Of the fourth payment.

8. What is the difference between 4½% of $740, and 2½% of $861?

9. If I purchase 729 Dl of brandy and lose 11% of it through leakage, how much will then be left me?

10. Add together 25% of $763v32, 16% of $847, and 6¼% of $12 346v17.

11. Brian Carroll dying leaves an estate worth $17 429 to be divided among 3 daughters, Catherine, Eloise, and Marguerite. Catherine is to get 43% of it, Eloise, 37%, and Marguerite the remainder.

a. What per cent does Marguerite get?
b. What is Catherine's share?
c. What is Eloise's share?
d. What is Marguerite's share?

12. A brick kiln contains 29 800 bricks and it is found, after burning, that 17% of them are worthless.

a. What per cent of them were good?
b. How many bricks were good?
c. How many were bad or worthless?

13. What is the commission on $1678v3 @ 2¼%?

14. An agent collects debts to the amount of $887. What is his commission @ 3⅓%?

15. A correspondent purchases teas for Michael Tarpy to the tune of $7193v16. What will be the correspondent's commission @ 2⅛%?

16. Michael Conway sells 718 barrels of flour for Mary Stachowski @ $7v13 a barrel. What will his commission amount to thereon at 4½%?

17. Harvey Hall as factor for Mary Fogarty disposes of 8243½ Hl of wheat @ $5⅛ per Hl. What does he earn getting 2% commission?

18. What is the brokerage on $7 893v87 @ 2%?

19. What is the brokerage on $8 000 @ ⅞ of 1%?

20. What is the brokerage on $19 788v3 @ 3¾%?

147. Thomas Nestor receives $16 783▿56 from Israel Aubin with instructions to deduct his commission (on the amount he will invest) @ 3½%, and invest the balance of the $16 783▿56 in sugar @ 17¾ c. per Cg.

a. How much sugar does he ship to Mr. Aubin?

b. What does it cost?

c. What does his commission amount to?

Operation with Explanation.

The commission on $1 paid out for sugar at the rate of 3½% is $▿035. Hence for every $1▿035 he receives *he* invests $1 in sugar and keeps 3½ c. Evidently, then, if we divide the $16 783▿56 received by $1▿035, (find how many times 1▿035 is contained in 16 783▿56), we shall know how often he invested $1 for sugar, i. e., how many dollars he so invested, and, at the same time, how many times 3½ c. he kept as commission.

3½% = 3▿5%;

3▿5 ÷ 100 = ▿035, the rate per unit;

$1 + ▿035 = $1▿035;

$16 783▿56 ÷ 1▿035 = $16 216, amount invested in sugar;

$$\left.\begin{array}{l}\$16\,783\triangledown56-16216,\text{ or}\\ \$16\,216\times\triangledown035\end{array}\right\}=\begin{array}{l}\$567\triangledown56,\text{ Mr. Nestor's commis-}\\ \text{sion;}\end{array}$$

16 216 ÷ ▿1775 = 91357▿$\dot{7}$464788732394366197183098591549295$\dot{7}$, making, say, 91 357 746 479 mg of sugar shipped.

148. EXERCISES.

1. John Conway received $4080 from Charles Robinson with instructions to purchase stock. After deducting his brokerage @ 2%,

a. How much money had he to invest?

b. What did his commission amount to?

2. Hugh Victor sends his factor $8088▿08 with instructions

to deduct his commission @ 4% and purchase lace with the balance. Find,

a. The factor's commission;
b. The sum paid for lace.

3. Warren Wilson receives $8470 with instructions to purchase flour @ $7 per Q. He is to get 5% on the amount he pays for the flour.

a. How many Mg of flour does he purchase?
b. What does it cost?
c. What does his commission amount to?

4. Patrick Conway receives $10894½ with instructions to invest it, deducting his brokerage therefrom first @ ⅞ of 1%. Find,

a. The amount he invests;
b. The amount of his brokerage.

5. Edouard Stampfler remits to his agent, Silas Fallass, $13000, instructing him to purchase broadcloth @ $3v6 per m. Mr. Fallass keeps 4% on the sum he invests as his commission.

a. How much does Mr. Fallass get?
b. How much does he invest?
c. How many Dm of cloth does he send to Mr. Stampfler?

6. Jacques Felix insures his house and furniture for $7300 at the rate of 1¼% per annum, (i. e., per year). What premium must he pay per year?

7. What would be the premium for insurance to the amount of $7500 @ 2%?

8. What would be the premium at the rate of ⅞ of 1% on an insurance policy for $837500?

9. A firm owning 4 steamers on Lake Ontario effect an insurance thereon with a company in Montréal to the amount of $27000 on each, paying 4½% premium. How much premium do they pay?

10. Charles Stone insures his barns, etc., to the amount of \$1780, paying $\frac{1}{2}$ of 1% premium. How much premium does he pay?

11. A vessel running between San Francisco and Los Angeles is insured for \$12 350, at the rate of $1\frac{1}{8}$% premium per mo. To what will the premium amount in 8 mo.?

149. To find what sum must be insured on property, so that, if destroyed, its value *and* the premium may both be recovered, we must divide the value of the property by \$1, less the premium on \$1, at the given rate per cent.

150. Illustrative Example.

John Pickler wishes to insure a vessel valued at \$19080, so, that, if it be wrecked, he may recover both its value and the premium. For what sum, then, must he insure @ $4\frac{3}{5}$%?

Operation and Explanation.

$4\frac{3}{5}$% = 4v6%;

4v6 ÷ 100 = v046, the per unit.

Now, if he insures goods to the value of \$1 @ $4\frac{3}{5}$% and they are destroyed, he gets only \$1 − \$v046 towards his loss since said \$v046 goes to reimburse him for the premium he had paid thereon. For every dollar of his loss, therefore, he receives only \$v954, since 1 − v046 = v954. As the recovery of \$v954, then, requires \$1 to be insured, the recovery of \$19080 will require as many dollars to be insured as v954 is found to go times into 19080, that is \$20000. I. e., 19080 ÷ v954 = 20000.

151. Exercises.

1. What sum must be insured on a house valued at \$6014 @ 3% so as to secure both the premium and the value?

2. Moise Blais owns a cargo of goods valued @ $19300. For what sum must he insure it, @ 3½%, so that, in case the whole is lost, he may recover both the value and the premium?

3. Paul Kruger owns buildings worth $22795. For what sum must he insure them @ 6% in order to cover both the said value and the premium?

4. What sum must be insured @ 2¼% on goods worth $1564, so that the owner be secured against *all* loss?

5. A steamer worth $32 987▽5 is insured at the rate of 5¾% for such sum as will cover, in case of a total loss, both its value and the premium. For what sum is it insured?

152. "Stock" is a term used to denote the capital of moneyed institutions. It is divided into equal parts called shares, most generally of $100 each.

The nominal or par value of a share is its *face* value.

Its market value is the sum or price at which it can be sold and is liable to rise or fall just as is the price of any other thing or commodity of value.

When stock sells for more than its face or par value, (par means "equal to"), it is said to be at a premium, when for less, at a discount.

This premium or discount is always reckoned at the per cent it may happen for the moment to be on the par value.

153. To find how much stock a given sum will purchase at a given time, we must divide the given amount or sum by the then market value of $1 face value of the stock in question. If the stock is at par, $1 will buy $1 of stock; if at a discount, the discount on $1 must first be subtracted from the one dollar before dividing; if at a premium, the premium on $1 must first be added to it.

154. ILLUSTRATIVE EXAMPLES.

How much stock @ 10% discount can be purchased for $25000?

Operation.

$1−$v1=$v9;
$25000÷v9=250000÷9=$27 777v$\dot{7}$, the answer.

How much stock @ 10% premium can be bought for $25000?

Operation.

$1+$v1=$1v1;
$25000÷1v1=250000÷11=$22 727v$\dot{2}\dot{7}$, the answer.

155. EXERCISES.

1. What amount of stock, i. e., stock of a face value to the amount of how much can Pierre Hamel buy for $8000 @ 12% premium?

2. Jean Baptiste Archambault owns $16400 stock of the bank of Montréal, and sells out @ 13% above par. What sum of money does he so get?

3. Hyacinthe Brissette invests $9000 in stock then selling @ 19% discount. What is the face of the stock he gets?

4. Chester Arthur invests $8700 in stock selling @ 11% premium. What amount of stock (face value) does he get?

5. Robert Lee remits to his agent, John McLean, $17499v24 with directions to deduct brokerage @ 1¼%, (on what will then be left *to* invest), and invest the balance in Grand Trunk Railway stock then selling @ 10% premium. What amount of stock (face value) does he secure?

6. Zénon Bleyenbergh receives $20200 with instructions to deduct his commission @ 1%, and invest the balance in stock then selling @ 3% discount. What amount of stock must he buy?

7. William Edwards owns 200 shares of bank stock, (each share's face value being $100), and the stock is at a discount of ½ of 1%. Andrew Gerup buys it through the agency of a stock-jobber to whom he has to pay ½ of 1%, (always on the amount he, the stock-jobber, pays for the shares), brokerage. How much will the 200 shares thus cost Mr. Gerup?

Evidences of indebtedness are bought in much the same way.

156. Duty is a certain designated charge made on the importation (or exportation) of certain specified articles into (or out of) the realm.

Duties are specific or ad valorem.

A specific duty is a fixed sum per unit levied on the goods without any reference to their value.

An ad valorem duty is simply so much per cent on the value (where purchased) of the goods.

157. To find the specific duty it is but necessary to multiply the sum or rate per unit of the duty fixed by the number of units invoiced.

158. Exercises.

1. What is the specific duty @ 47 c. per m on 214 Dm of black silk velvet?

2. What is the specific duty @ 13 c. per Dl on 129 Hl of oil?

3. What will be the ad valorem duty, @ 21%, on an invoice of silk which cost $11749▿28?

4. What is the ad valorem duty, @ 7½%, on 40 boxes of tea which cost $74▿41 per box?

5. What is the specific duty, @ $1▿2 per 100 Cg, on 11 bags of rice, each weighing 7 Mg?

6. What is the duty on an invoice of 151 pairs of boots and shoes which cost $3▿01 a pair, the duty being an ad

valorem duty of 20% *and* a specific duty in addition of 15 c. a pair?

7. A tax of \$86 895 is to be levied in a county having ratable property to the value of five million seven hundred and ninety-three thousand dollars.

a. What will be John Hanlon's tax, whose property is valued at \$18600?

b. What will be Robert Crame's tax, whose property is valued at \$7500?

c. What will be Andy Rock's tax, whose property is valued at \$11400?

159. Interest is money paid or allowed for the use of money. It, too, is computed at some certain given rate per cent, (per annum). *Time*, then, is an additional factor to be taken into account in computing interest.

The sum being used (owing) is called the principal; the "rate per cent" is (in interest) the sum to be paid for the use of each \$100 for one year; the rate per unit is the same on \$1; the interest is the whole sum received or to be received for the use of the principal for the entire time; and the amount is the principal and interest added together.

Interest problems generally can be solved by the Rule of Three or by *practice*.

160. Let $P =$ Principal, $I =$ Interest, $r =$ rate *per unit*, and $t =$ time (in years).

Then we will have the following

RULE:

a. $I = Prt$. I. e., Interest = Principal × rate × time.

And this being so, it necessarily follows that either of the other 3 quantities will be equal to what we will get when we multiply together the remaining 2 and divide their product into the interest.

Therefore from this first formula, $I = P\,r\,t$, follow the following 3, To Wit:

$$\left.\begin{array}{l} \beta.\ P=\dfrac{I}{r\,t}; \\ r=\dfrac{I}{P\,t}; \\ t=\dfrac{I}{P\,r}. \end{array}\right\}$$

Substitute *any* 4 numbers for I, P, r, and t; say 100, 10, 5, and 2 respectively.
10×5×2=100.
Hence 100÷(5×2)=10;
100÷(10×2)=5; and
100÷(10×5) will be equal to 2.

So by remembering the first of these 4 formulæ we really have all 4, and can therefore find the 4th term in almost any practical interest problem, the other 3 having been given.

161. ILLUSTRATIVE EXAMPLES.

What is the interest on $342v2 for 7 y. and 8 mo. @ 8% ?

Operation.

8÷100=v08, therefore 8%=v08 per unit;
7 y. and 8 mo.=7v6̇ y., an inconvenient multiplier.

6 mo.=½ y.	2	$342v2
		7
		2395v4
2 mo.=⅓(of 6 mo.)	3	171v1
		57v03̇
		2623v53̇×v08=$209v8826̇

EXPLANATION.

1 times 342v2 is 342v2, and as 6 mo. is ½ a y., 6 months' times, (i. e., ½ a time) 342v2 will be 171v1. And as 2 mo. is ⅓ of 6 mo., 2 months' times 342v2 will be ⅓ of ½ of it. Adding these 2 parts to 7 times the 342v2 gives us 2623v53̇, which, when multiplied by the v08 gives us $209v8826̇, the required answer.

What principal will produce $207v5 interest in 6½ y. @ 4¾%?

Operation.

4¾=4v75; and
4v75÷100=v0475;
6½=6v5;
v0475×6v5=v30875;
207v5÷v30875=20750000÷30875;

This will give a quotient or answer of $672v06, since what is over amounts to less than half a cent. And in reckoning money, 5 mi. or over in the *final* answer is counted another cent, whilst less is disregarded. The *exact* quotient would be $672\mathrm{v}\dot{0}6477732793522267\dot{2}$.

At what rate pet cent will $729v18 give $113v428 interest in 9 y. and 4 mo.?

Operation.

9 y. and 4 mo.=9⅓ y.

4 mo.=⅓ y.	3	729v18
		9
		6562v62
		243v06
		6805v68

$113\mathrm{v}428 \div 6805\mathrm{v}68 = 11342\mathrm{v}8 \div 680568, = \mathrm{v}01\dot{6}$;
$\mathrm{v}01\dot{6} \times 100 = 1\mathrm{v}\dot{6}$, wherefore the answer is 1⅔%.

In what time will $850 yield $88v4 interest @ 13%?

Operation.

13÷100=v13;
850×v13=110v5;
88v4÷110v5=884÷1105,=v8;
v8×12=9v6;
v6×30=18; hence 9 mo. and 18 d. will be the required answer.

NOTE.—*If in the time found in the final answer there is any fraction of a day over, it must be counted as another full day.*

162. EXERCISES.

1. What is the interest on $723▿19 for 7▿32 y. @ 6▿7 % ?
2. To what sum will $857▿19 amount in 6½ y. @ 6½ % ?
3. In what time will $654▿ yield $294▿3 interest @ 9 % ?
4. At what rate per cent will $2700 amount to $3210 in 5 y. ?
5. In what time will $270 yield $87 interest @ 7 % ?
6. To what sum will $680 amount in 11½ y. @ 11% ?
7. Find the interest on $679▿18 @ 7¾% for 11▿5 y.
8. At what rate will $950 amount to $1757▿5 in 10 y.?
9. In what time will $666 amount to $1347▿5 @ 6% ?
10. In what time will $273 yield $100 interest @ 9 % ?
11. What principal will produce $45▿032 interest in 2 y. @ 5% ?
12. At what rate will $749▿4 yield $262▿29 interest in 7 y.?
13. Find the interest on $167▿47 @ 11% for 9 y.
14. What is the interest on $6 @ 1% for 7 y.?
15. Find the interest on $1463 for 3 y., 11 mo., and 12 d. @ 7%.
16. Find the interest on $189▿7 for 5 y. and 7 mo. @ 6%.
17. Find the interest on $146327▿94 for 3 y. and 2 mo. @ 6%.
18. Find the interest on $28 967▿85 for 11 y. @ 5%.
19. What is the interest on $763▿22 @ 4% for 6 y.?
20. Find the interest on $573▿3 @ 7% for 3 mo. and 13 d.
21. What is the interest on $14278▿75 @ 8% for 2 y., 11 mo., and 11 d.?
22. What is the interest on $99 999 @ 3% for 9 y., 9 mo., and 9 d.?
23. What is the interest on $69▿88 for 3 y., 4 mo., and 27 d. @ 9% ?
24. Find the interest on $742▿36 @ 12% for 3 y. and 28 d.
25. To what sum will $200 amount in 7 y., 4 mo., and 11 d. @ 7¼% ?
26. To what sum will $743▿63 amount in 9 y., 3 mo., and 9 d. @ 2⅔% ?

27. Find the interest on $1 for 17 y., 11 mo., and 15 d. @ $5\frac{4}{5}$%.

28. Required the interest on $4576▿89 @ $4\frac{1}{6}$% for 2 y. and $1\frac{5}{6}$ mo.

29. Required the principal that will draw $2 583▿278 interest in 7 y., 10 mo., and 6 d. @ 8%.

30. Find the interest on $715▿3 for $3\frac{1}{4}$ y. @ 3%.

31. To what sum will $5 555▿55 amount in 5 y., 5 mo., and 5 d. @ 5%?

32. At what rate per cent will $2479 yield $371▿85 interest in $2\frac{1}{2}$ y.?

33. What is the interest on seven hundred and eighty-four dollars from May 9th to December 11th of the same year @ 6%?

163. If partial payments have been made on a debt before the date of final settlement,

α. Compute the entire interest on the whole debt at the given fixed rate for the entire time up to the date of such final settlement and add it to said principal or entire debt;

β. Compute the interest likewise at the said given rate on each several payment consecutively from the day on which it was made to the said date of final settlement and add it to such payment;

γ. Add together all these so-augmented payments and subtract their sum from the amount found as per *α*.

164. ILLUSTRATIVE EXAMPLE.

BOWDLE, S. D., October 18th, A. D., 1891.

$554.

Three months after date I promise to pay to the order of John de Breboef five hundred and fifty-four dollars with interest @ 6% per annum.

ERMINIE NORMANDIN.

On this note, at the date of settlement, September 12th, A. D. 1894, are found the following endorsements, to wit:—

"November 24th, '91, paid hereon $17v7";
"April 3d, '93, " " $100";
"October 18th, '93, " " $48v88."

What amount was yet due and to be paid on said note on said date of final settlement?

Operation.

September is the ninth month of the year and October the tenth.

1894 " 9 " 12
1891 " 10 " 18

2 y. " 10 mo. " 24 d.=2v9 y. The time.

554+(554×2v9×v06)=650v396, 1st amount in dollars.

1894 " 9 " 12
1891 " 11 " 24

2 y. " 9 mo. " 18 d.=2v8 y. Time.

(17v7×2v8×v06)+17v7=20v6736, amount in dollars of the 1st payment.

1894 " 9 " 12
1893 " 4 " 3

1 y. " 5 mo. " 9 d.=1v441$\dot{6}$ y. Time.

(100×1v441$\dot{6}$×v06)+100=108v65, amount in dollars of the 2d payment.

1894 " 9 " 12
1893 " 10 " 18

" 10 mo. " 24 d.=v9 y. Time.

(48v88×v06×v9)+48v88=51v51952, amount in dollars of the 3d payment.

	20v6736
108	65
51	51952
180	84312

650v396
180v84312
469v55288

Balance to be paid upon final settlement would therefore be $469v55.

165. EXERCISES.

1. Find the amount still due September 22d, A. D., 1895, on the following note.

GUELPH, ONTARIO, CANADA, June 22d, 1888.
$211v88.

I promise to pay on demand to Alexandre Kennedy the sum of two hundred and eleven dollars and eighty-eight cents with interest @ 7% from the date hereof.
JOHN WOLLOWSKI.

Endorsed thereon.

"October 31st, '90,	paid hereon	$11v11";
"February 29th, '92,	" "	$91";
"June 24th, '93,	" "	$14v14";
"July 30th, '93,	" "	$25";
"July 18th, '95,	" "	$50."

2. Find the amount due August 15th, A. D., 1894, on the following note.

ANN ARBOR, MICHIGAN, August 15th, 1890.
$300.

On or before the first day of January, A. D., 1892, I promise to pay to the order of Thomas Wetherald the sum of three hundred dollars with interest @ 15½%.
ANDRÉ STEFFLRE.

Endorsements.

"December 26th, '90, paid hereon $70";
"January 13th, '94, pd. hereon $35."

3. Find the amount or balance due on the following note on January 1st, A. D., 1898.

ABERDEEN, S. D., April 1st, A. D., 1877.

$991v89.

On or before the first day of February, A. D., 1888, I promise to pay to the order of Jennet Hughes the sum of nine hundred and ninety-one dollars and eighty-nine cents with interest @ 4½%.

RICHARD ROE.

Endorsements thereon.

"April 2d, '79, pd. hereon $89v19";
"May 1st, '88, " " $42v18";
"August 14th, '88, " " $3";
"June 19th, '91, " " $14v15";
"January 22d, '93, " " $500";
"March 17th, '94, " " $51v33";
" " 11th, '96, " " $100."

166. If the interest is to be compounded, we must compute it up to the time when interest is first to be paid, (to the end of one year unless otherwise stipulated for), add it to the principal, and then take the thus augmented principal for a new principal on which to again compute the interest at the given rate from said last time when interest was first to be paid up to the time when it is next to be payable. We must now add the interest so last found to said (new) principal so last used and then compute on *this* sum, adding to it the interest again when found, and so on to the time of payment.

DISCOUNT.

167. Discount is an allowance made for payment of a debt before it becomes due.

The present worth of a debt or demand payable at some future time without interest is that sum of money which, being put out at legal interest, *will amount to the debt or demand* by the time said debt becomes due. It is again the principal which will, at the rate money is just then worth, amount to the given sum (the debt) in the time yet to intervene before payment.

168. RULE.

$P=\frac{A}{1+rt}$. I. e.,

α. Find the interest on $1 at the given rate for the given time;

β. Add this to the $1;

γ. Divide their sum into the debt.

169. ILLUSTRATIVE EXAMPLE.

What is the present worth of a note for $860 payable 3 y. hence without interest, legal interest being 7%?

Operation.

1×▿07×3=▿21, interest on $1 is therefore $▿21;

$1+$▿21=$1▿21, amount of $1 for the given time at the given rate;

860÷1▿21=86000÷121,=710▿74, which will be dollars.

170. EXERCISES.

1. Required the present worth of $962 payable in 1 y., the use of money being worth 1¼% per annum.

2. What is the present worth of a debt of $2212v2, due in 5¾ y., legal interest being 3½%?

3. Find the present value of $1003v35 due in 7 mo., legal interest being ⅛ of 1%.

4. What is the present worth of $916 due in 7 y. and 2 mo., legal interest being 15%?

5. What sum would to-day equitably pay a debt due 2 y. hence and amounting to $54231, interest being 5⅒%?

6. What is the present worth of $4306v16 due in 1 y. and 5 mo., legal interest being 6%?

7. Find the present worth of a note for $1674 due in 11 mo. (without interest), legal interest being 12½%.

8. A merchant owes $444v4 payable in 1⅔ y., and also $968v05 due in 2⅓ y. What sum would *now* pay both these debts, legal interest being 7%?

9. What is the discount @ 7% on $3024, due, ⅑ of it in 6 mo., the balance in 10 mo.?

171. Banks, in addition to this discount, are suffered to take interest also on the discount they keep back. They add 3 days to the time the debt has yet to run, and then compute the interest on the entire face of the debt for this thus lengthened time. This makes it simple interest for all purposes of computation, pure and simple.

Partnership, profit and loss, and exchange problems are but examples in proportion, the last-named in conjoined proportion.

EQUATION OF PAYMENTS.

172. If it is desired to pay in one payment 2 or more debts due at different times, we must,

α. Reduce all the several "times" to the same unit, if not already so;

β. Multiply each several debt by the time in these units it has yet to run;

γ. Add together the products thus secured;

δ. Add the several debts all together;

ε. Divide the sum of these debts into that of the said products.

173. ILLUSTRATIVE EXAMPLE.

January 1st, A. D. 1898, Thomas Ward purchases of George Steffire a vessel for $7000. He is to pay $2000 in 3 mo., two thousand dollars in 5 mo., and the balance in 11 mo., without interest. When should he pay if he pays all at once?

Operation.

$2000× 3=$ 6000;
$2000× 5=$10000;
$3000×11=$33000.
$7000. $49000.

49000÷7000=7.

He could, therefore, equitably pay up August 1st, 1898.

174. EXERCISES.

1. Clara Franc owes John Hovey $20 payable in 6 mo., $50 payable in 1 y., and $90 payable in 8 mo. In what time from now can all be paid in one payment without loss or undue gain to either party?

2. Erminie Normandin owes $450 which she is to pay as follows, To Wit: $100 forthwith, $300 in 4 mo., and the balance in 6½ mo. Find the equated time of payment.

3. Jennet Hughes owes Caroline Emmot $600 and is to pay $200 in 3 mo., $150 in 4 mo., and the balance in 6 mo. When could it all be paid in one payment in equity to both?

4. A debt is to be discharged in the following manner: ¼ down, ¼ in 3 mo., ¼ in 6 mo., and ¼ in 9 mo. Find the equated time of payment.

5. James Davies owes $125 which is due as follows: $50 in 2 mo., $40 in 5 mo., and the balance in 7 mo. When could he in fairness pay it all at once?

6. Bought of Robinson, Straus & Co. goods in the following amounts, on a 6 months' credit:

January 15th, A. D. 1900, a bill of $750;
February 1st, “ “ “ “ “ “ $201;
March 5th, “ “ “ “ “ “ $109;
April 20th, “ “ “ “ “ “ $20;
June 1st, “ “ “ “ “ “ $664; and
“ 18th, “ “ “ “ “ “ $193v56.

July 1st (of the same year) I wish to give my note for the total amount. On what day should this note become due and payable without interest and without grace?

INVOLUTION AND EVOLUTION.

175. Involution is the process of finding the power, (1st, 2d, 3d, 4th, 5th, etc., etc., ad libitum), of a given number by multiplying it by itself, taking it as a factor, one or more times. Every number is itself the first power of itself. If it be taken twice as factor, multiplied once by itself, we get its 2d power or square; if taken thrice as factor, we get its 3d power or cube; if so taken 4 times, we get its 4th power, and so on.

By evolution, then, we find the root of a given power.

The terms "*power*" and "*root*" are therefore correlative. If one number is a power of another, then that other is the corresponding root of the former. The 2d power of a number is also known as its square; the 3d, as its cube; and these numbers, then, are the square root and cube root respectively of these powers.

A small figure written to the right of a number and a little higher up is used to indicate how often that, as a root, is to be taken as factor in order to produce (*make*) the required power. It is called the index or exponent of the power. Thus,

$3^1=3,=3,=$1st power of 3, called three;

$3^2=3\times3,=9,=$2d power of 3, called three square;

$3^3=3\times3\times3,=27,=$3d power of 3, called three cube (or the cube of 3);

$3^6=3\times3\times3\times3\times3\times3,=729,=$6th power of 3, called three in the sixth power.

$(2+4)^2=2+4$, (i. e., $6)^2,=6\times6,=36$; but

$2^2+4^2=2\times2$, (i. e., 4) $+4\times4$, (i. e., 16).

Hence $2^2+4^2=4+16,=20$.

Fractions are involved by so multiplying both terms. Thus,

$(\frac{3}{4})^2=\frac{3\times3}{4\times4}$. But $3\times3=9$, and $4\times4=16$. Therefore $(\frac{3}{4})^2=\frac{9}{16}$.

$(\frac{7}{8})^3=\frac{7\times7\times7}{8\times8\times8}, =\frac{343}{512}$.

Mixed numbers should be reduced to their equivalent fractions or plain numbers before being involved.

176. Illustrative Example.

Find the cube of $1\frac{1}{2}$.

Operation.

$1\frac{1}{2}=\frac{3}{2}$.

$(\frac{3}{2})^3=\frac{3\times3\times3}{2\times2\times2}, =\frac{27}{8}, =3\frac{3}{8}, =3\triangledown375$.

177. Exercises.

1. Find the 7th power of 5.
2. Find the cube of 14.
3. What is the 6th power of 2?
4. Find the 5th power of $3\frac{5}{8}$.
5. What is the square of $\frac{3}{5}$?

178. Let it be required to multiply 5^2 by 5^3.

$5^2=5\times5$;

$5^3=5\times5\times5$; wherefore,

$(5^2\times5^3)=(5\times5)\times(5\times5\times5)$; which is the same as, $5\times5\times5\times5\times5$, and is equal to 5^5.

This is 5^{2+3}; hence,

2 or more powers of a given number are multiplied together by merely adding together their indices or exponents. $4^1\times4^1=4^2$.

179. Let it be required to divide 4^4 by 4^3.

$4^4=4\times4\times4\times4$;

$4^3=4\times4\times4$; wherefore,

$4^4\div4^3=(4\times4\times4\times4)\div(4\times4\times4)$.

Canceling all factors equioften common to both terms, (see Point 58), leaves us one 4 in the dividend.

$256 \div 64 = 4, = 4^{4-3}, 4^1, = 4.$ But,

$256 = 4 \times 4 \times 4 \times 4;$ and

$64 = 4 \times 4 \times 4.$

To divide one power of a number, then, by another power thereof, subtract the exponent of the divisor from that of the dividend.

180. Let it be required to find the cube of 7^2.

$(7^2)^3 = 7^2 \times 7^2 \times 7^2, = (7 \times 7) \times (7 \times 7) \times (7 \times 7), = 7 \times 7 \times 7 \times 7 \times 7 \times 7.$

$7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6, = 7^{2 \times 3}.$

So to find any required power of a given power of a number, multiply the exponent of the given power by that of the required power.

181. EXERCISES.

1. Find the 4th power of 2▿045.
2. Required the 4th power of $4\frac{2}{9}$.
3. Multiply together 4^3, 4^{11}, 4^4, and 4^7.
4. Divide 12^2 by 12.
5. Divide 18^9 by 18^5.
6. Divide 6^6 by 6^1.
7. Find the 7th power of 7^7.
8. Find the 18th power of $(3\frac{1}{2})^3$.
9. $\{(8 \times 8^2) \div (8 \times 8)\}^7 = ?$
10. What is the 3d power of $\frac{2}{5}$ of $7\frac{1}{4}$?
11. Required the square of $8 \div (\frac{2}{7} \div 1\frac{1}{2})$.

182. A (corrupted) form of the letter r, called the radical sign, (from radix, a root), is placed before a number to express its root, and whenever this root is to be a root higher than the *square* root, it is expressed by a small figure placed in this sign. Thus,

$\sqrt{4}$ = the square root of 4;
$\sqrt[3]{195}$ = the cube root of 195;
$\sqrt[12]{201\,764}$ = the twelfth root of 201 764;
$\sqrt[7]{5\frac{1}{2}}$ = the seventh root of $5\frac{1}{2}$;
$\sqrt{314^4}$ = the square root of 314 in the fourth power;
$\sqrt[5]{\frac{7}{8} \text{ of } 1\frac{1}{4}}$ = the fifth root of $\frac{7}{8}$ of $1\frac{1}{4}$;
$\sqrt[3]{12\text{v}305}$ = the cube root of 12v305.

183. A fractional exponent is sometimes used instead of said radical sign. If the number whose root is to be thus expressed is in its first power, the dividend of said fractional index will be a 1, if it is in its second power, said dividend will be a 2, and so on. Hence the roots above can also be written $4^{\frac{1}{2}}$; $193^{\frac{1}{3}}$; $201\,764^{\frac{1}{12}}$; $5\frac{1}{2}^{\frac{1}{7}}$; $314^{\frac{4}{2}}$; $(\frac{7}{8}\times 1\frac{1}{4})^{\frac{1}{5}}$; $12\text{v}305^{\frac{1}{3}}$, the divisors of said fractional exponents expressing the root. Since, now, all factors common to both terms of any fraction may be eliminated without affecting the quotient, the square root of the above $314^{\frac{4}{2}}$ will be 314^2. (See Point 58.)

$\sqrt{314^4} = 314^{\frac{4}{2}}$. Canceling this fraction by 2 gives us $314^{\frac{2}{1}}$. But $2 \div 1$, (i. e., $\frac{2}{1}$), $= 2$, wherefore $314^{\frac{2}{1}} = 314^2$, which is $314^{4 \div 2}$. Hence,

To extract any root of a given *power* of a number, divide the index of the power by the exponent of the root sought.

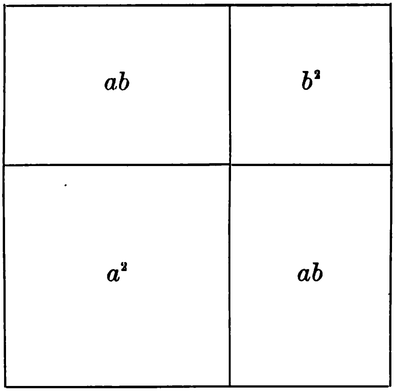
184. EXERCISES.

1. Extract the 8th root of 3^{40}.
2. Extract the 9th root of 9^{18}.
3. Extract the cube root of 5^3.
4. Find the square root of $11^{\frac{12}{1}}$.
5. Find the 4th root of 21^{20}.
6. What is the 6th root of $(\frac{1}{2} \text{ of } \frac{1}{3})^{18}$?

185. To extract the square root of a number is to find that number which, when multiplied once by itself, will produce the given number.

Let a here = the tens, and b = the units of any supposed given number.

Then $(a+b)^2=a^2+2ab+b^2$,



Hence the

186. RULE.

α. If the given number has an odd number of figures to the right of the v, annex a cipher;

β. Beginning at the v, divide off the number in both directions into periods of 2 figures each by writing a curve over each of such periods;

γ. Draw a horizontal line above the number;

δ. Find the highest square contained in the left-hand period thus formed, placing it under, and its root over, said period;

ϵ. Subtract this square from said period and annex the next period to the remainder for a dividend, writing a curve before it;

ζ. Double the part of the root found and place before this curve for a trial or partial divisor;

η. Find how often this trial divisor will go into this thus constructed dividend, (keeping in view its completion), exclusive of the right-hand figure, and place the figure expressive of such number of times to the right of it as also in the root above;

θ. Multiply the now thus completed divisor by this last figure so put into the root, subtracting the product from the (last used) dividend, and to the remainder bring down (affix) the next period for a new dividend;

ι. Again double the part of the root found for a new trial divisor, and proceed over again as above, and so on to the end;

κ. If the number is a mixed number or any kind of a fraction, reduce it to its equivalent simple fraction in its lowest terms, then, if both terms be exact powers of the root sought, extract their respective roots as desired separately, otherwise change it first to a plain number.

187. Illustrative Examples.

Find the square root of 21 455 424.

Operation.

```
       )4632, Required root.
       ‾‾‾‾‾‾‾‾‾‾‾
        ⁀ ⁀ ⁀ ⁀
        21455424v
        16 x x x
   86 | 545
      | 516
      ‾‾‾‾‾‾‾‾
  923 | 2954
      | 2769
      ‾‾‾‾‾‾‾‾
 9262 | 18524
      | 18524
      ‾‾‾‾‾‾‾‾
            0
```

Find the square root of 56 294v362.

Operation.

)237v2643293+ approximate answer.

	56294v3620
	4
43	162
	129
467	3394
	3269
4742	12536
	9484
47446	305220
	284676
474524	2054400
	1898096
4745283	15630400
	14235849
47452862	139455100
	94905724
474528649	4454937600
	4270757841
474528658	18417975900, etc., etc.

Find the square root of 121.

Operation.

)11, Answer.

121▿

1

21 | 21

  | 21

0

Find the square root of ▿00076.

Operation.

▿00076=▿000760.

)▿02756+ ad libitum aut infinitum.

▿000760

4 ×

47 | 360

  | 329

545 | 3100

  | 2725

550 | 37500

Find the square root of 2¼.

Operation.

$2\frac{1}{4}=\frac{9}{4}$. $\sqrt{\frac{9}{4}}=\frac{3}{2},=1\frac{1}{2}$.

Find the square root of $\frac{4}{121}$.

Operation.

$$\frac{\sqrt{4}}{\sqrt{121}} = \frac{2}{11}$$

Extract the square root from $\frac{7}{8}$ of $\left\{\left(\frac{1}{8} \div 9\right) + \frac{1}{2}\right\}$.

Operation.

$\frac{1}{8} \div 9 = \frac{1}{8} \div \frac{9}{1}, = \frac{1}{8} \times \frac{1}{9}, = \frac{1}{27};$

$\frac{1}{27} + \frac{1}{2} = \frac{2}{54} + \frac{27}{54}, = \frac{29}{54};$

$\frac{7}{8}$ of $\frac{29}{54} = \frac{7}{8} \times \frac{29}{54}, = \frac{203}{482}, = \nabla 4699\dot{0}7\dot{4}.$

$$\begin{array}{r|l}
\multicolumn{2}{l}{\underline{)\nabla 6854979 + \text{ ad infinitum. The number is not a square.}}} \\
\multicolumn{2}{l}{\quad \nabla\overset{\frown}{46}\overset{\frown}{99}\overset{\frown}{0\dot{7}}\overset{\frown}{4\dot{0}}} \\
\multicolumn{2}{l}{\qquad 36 \quad \times \;\; \times \;\; \times} \\
128 & 1099 \\
 & \underline{1024} \\
1365 & 7507 \\
 & \underline{6825} \\
13704 & 68240 \\
 & \underline{54816} \\
137089 & 1342474 \\
 & \underline{1233801} \\
1370987 & 10867307 \\
 & \underline{9596909} \\
1370994 & 127039840
\end{array}$$

188. To extract the cube root of a number is to find a number which, when multiplied by its own square, will make the given number.

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$, whence the

189. RULE.

α. If the number of figures to the right of the v is not a number exactly measurable by 3, affix 0's enough to make it so;

β. Beginning at the v, divide the number off in both directions into groups or periods of 3 figures each, by writing a curve over each separate group;

γ. Find the highest cube contained in the left-hand period thus formed and place it under, and its root over, such period;

δ. Subtract this cube from said period and to the remainder annex the next period for a dividend, i. e., carry said remainder to the next period;

ε. Multiply the square of the part of the root found by 300 for a trial divisor;

ζ. Find how often this trial divisor will go into this dividend, keeping in view its completion;

η. Add, now, to the trial divisor,

a. The part of the root previously found × the last figure placed in the root × 30,

b. The square of said last figure placed in the root;

θ. Multiply the thus completed divisor by the said last figure placed in the root, subtract the product from the dividend just used, and again carry the remainder for a new dividend to the *next* period as before;

ι. Again multiply the square of the part of the root already found by 300 for a new trial divisor, and so on as before to the end;

κ. If the number is a fraction or a mixed number, reduce it first to its equivalent simple fraction in its lowest terms if these will then be perfect cubes, and then extract their respective cube roots severally, otherwise reduce the number first to a plain number;

λ. Having placed each figure of the root immediately over the period used in finding it, the ∇ will, as well here, as in square root, come into position immediately over where found in the original number or power.

190. ILLUSTRATIVE EXAMPLES.

What is the cube root of 429 172 932 007?

Operation.

$$\begin{array}{r|l}
 &)7\quad 5\quad 4\quad 3,\ \text{Answer.} \\ \hline
 & \overset{\frown}{429}\overset{\frown}{172}\overset{\frown}{932}\overset{\frown}{007}\triangledown \\
 & \qquad\ \times\quad \times\quad \times \\
 & 343 \\ \hline
7^2\times300=14700 & 86172 \\
7\times5\times30=\ \ 1050 & \\
5\times5=\ \ \ \ 25 & \\ \hline
15775 & 78875 \\ \hline
75^2\times300=1687500 & 7297932 \\
75\times4\times30=\ \ \ 9000 & \\
4\times4=\ \ \ \ 16 & 6786064 \\ \hline
1696516 & 511868007 \\ \hline
754^2\times300=170554800 & \\
754\times3\times30=\ \ \ 67860 & \\
3\times3=\ \ \ \ 9 & \\ \hline
170622669 & 511868007 \\ \hline
 & 0
\end{array}$$

Required the cube root of $3\frac{3}{8}$.

Operation.

$$3\frac{3}{8}=\frac{27}{8}.$$

$$\sqrt[3]{\frac{27}{8}}=\frac{\sqrt[3]{27}}{\sqrt[3]{8}},=\frac{3}{2},=1\frac{1}{2}.$$

What is the cube root of $\frac{3}{5}\times1\frac{1}{4}$?

Operation.

$$\frac{3}{5}\times1\frac{1}{4}=\frac{3}{5}\times\frac{5}{4},=\frac{3}{4},=\nabla75,=\nabla750.$$

$$\underline{)\nabla9085+,\text{ ad libitum.}}$$

$$\begin{array}{r|l}
 & \nabla\overset{\frown}{750} \\
 & \underline{\;729} \\
9^2\times300=24300 & 21000 \\
9\times0\times30=\quad\;0 & \quad\;\;0 \\
0\times0=\quad\;\underline{\quad0} & \\
24300 & \\
\hline
90^2\times300=2430000 & 21000000 \\
90\times8\times30=\quad21600 & \\
8\times8=\quad\underline{\qquad64} & \\
2451664 & 19613312 \\
\hline
908^2\times300=247339200 & \;1386688000
\end{array}$$

191. Any root whose index has no prime factors other than 2 or 3, or 2 and 3 may be extracted by means of square and cube root in the following manner, To Wit:

α. Resolve the index of the root sought into its prime factors;

β. Then extract the root denoted by the first of these factors;

γ. Extract, thereupon, of this root, the root denoted by the next of said prime factors, and so on till all the prime factors of the root sought will each have been once so used.

192. ILLUSTRATIVE EXAMPLE.

What is the 6th root of 2 176 782 336?

Operation.

$6=3\times2;$

$\sqrt[3]{2176782336}=$ 1296, and

$\sqrt{1296}=$ 36, Answer.

or

$\sqrt{2176782336}=46656$, and

$\sqrt[3]{46656}=$ 36, Answer.

193. These as well as any and all other roots of *exact* powers can also be found by dividing the number, (power) by one of its (prime) factors, then dividing the quotient in like manner, and so on until the number is resolved into its prime factors, then grouping these prime factors into as many equal groups as is the number indicated by the index of the root sought.

194. ILLUSTRATIVE EXAMPLES.

What is the cube root of 429 172 932▿007?

Operation.

429 172 932▿007 ÷ 19 = 22 588 049▿053;
22 588 049▿053 ÷ 19 = 1 188 844▿687;
1 188 844▿687 ÷ 19 = 62 570▿773;
62 570▿773 ÷ 397 = 157▿609;
157▿609 ÷ 397 = ▿397, giving as prime factors:
19, 19, 19, 397, 397, and ▿397; Or
(19×397), (19×397), (19×▿397), which is equivalent to
(19×39▿7), (19×39▿7), (19×39▿7).
But 39▿7×19 = 754▿3, the required (cube) root.

What is the 5th root of 161 051?

Operation.

161 051 ÷ 11 = 14 641;
14 641 ÷ 11 = 1331;
1331 ÷ 11 = 121;
121 ÷ 11 = 11, giving us as factors 11, 11, 11, 11, and 11.
Wherefore 11 is the 5th root of 161 051.

195. Let us keep in view the following descriptions and principles, To Wit:

α′. A triangle is a plane figure having for its sides 3 straight lines, ergo having 3 angles.

β′. When one of its angles is just equal to the sum of the other 2 it is a right angle, (¼ of a c.), and the triangle is then said to be a right-angled triangle.

γ′. In a right-angled triangle the side opposite the right-angle is called the hypotenuse, and the sides containing it the base and the perpendicular respectively.

δ′. The square described on the hypotenuse of a right-angled triangle is always equal to the sum of the squares described on the other 2 sides thereof.

ε′. If a number is not an exact square it cannot have an exact square root, nor can it have an exact cube root unless it be an exact cube, but the further the "root" is run out the nearer we get to correctness in the answer we *do* get in either of these cases.

ς′. If the square of the radius of a circle be multiplied by 3v14159265+, the product will be the area nearly, and vice versa.

ζ′. If the diamètre of a circle be multiplied by 3v14159265+, the product will be the circumference, (nearly), and vice versa.

η′. If the circumference of a circle be multiplied by half the radius, the product will be the area.

θ′. If the square root be extracted from double the square of the radius of a circle, it will be one side of an inscribed square.

ι′. The altitude of a triangle is the length of a line drawn from the angle opposite, perpendicular to, and to the base (extended if need be).

ια′. The base of a triangle multiplied by half its altitude gives us its area.

ιβ′. Double the area of a triangle divided by either of its 2 dimensions, (base or altitude) will give the other of said 2 dimensions.

ιγ′. The square root of the product of any 2 numbers is a mean proportional between them.

ιδ′. Similar figures whether plane or solid are such as have the same form, and their like or corresponding dimensions are therefore proportional.

ιε′. The areas of the corresponding surfaces of similar figures are to each other as the squares of their like dimensions and vice versa.

ιϛ′. A sphere is a body bounded by a surface in which all the points are equidistant from a point within it called the centre.

ιζ′. The diamètre of a sphere is a straight line passing through the centre and terminating at both ends in the surface; and half of it is a radius.

ιη′. The circumference of a sphere is that of a circle having the diamètre of the sphere for its diamètre. It is known as a *great circle* in contradistinction to all other circles not circumferences that can be described on the sphere.

ιθ′. The diamètre of a sphere multiplied by its circumference is equal to its surface.

κ′. The surface of a sphere multiplied by $\frac{1}{6}$ of its diamètre is equal to its volume.

κα′. Spheres, cubes, and all other regular similar solids are to each other as the cubes of their like dimensions, and vice versa.

κβ'. If the number indicative of the number of places to the right of the v in a plain number be an odd number, i. e., a number *not* exactly measurable by 2, said plain number cannot be a perfect square.

κγ'. If the number indicative of such number of places be not exactly divisible by 3, the said plain number cannot be a perfect cube.

κδ'. The author is grieved to be still unable to give the exact ratio between the circumference and the diamètre of a circle. For ordinary purposes $3\frac{1}{7}$ is used and gives an answer so nearly correct as to satisfy many cases tolerably well — 3v14159265 will be sufficiently nearly exact for almost any purpose, and is the ratio employed in this work. But even this is not wholly exact except for as far as it goes. Though this decimal has been run out to 150 places, it still failed to end; nor is the end known.

κε'. π is used as a short, convenient character to represent this ratio. Thus,

π=3v14159265+;
πD.=circumference of a c.;
πr.^{2}=area " " ";
πD^2=surface of a sphere;
$\frac{\pi}{6}$D^3=volume " " "

κς'. A circle is a plane figure contained by one line so drawn and situated that all straight lines drawn from any and all point or points in it to a certain fixed point within the figure are constantly equal in length.

κζ'. The line so enclosing a circle is called its circumference.

κη'. The said fixed point in the circle is known as the centre of the circle.

$\kappa\theta'$. A straight line from the circumference (surface) of a sphere or from the circumference of a circle to its centre is called a radius thereof.

λ'. 2 radii in line constitute a diamètre.

196. Exercises.

1. $(7\triangledown 2 - \sqrt{27\triangledown 04})^5 \div (\frac{2}{3})^2 = ?$

2. Garcia Moreno has a rectangular field 208 Dm long and 1▿3 Hm wide. Required the length of one side of a square of like area.

3. If it cost $112 to enclose a field 216 m long and 24 m wide, what will it cost to so enclose a *square* field of equal area?

4. If a wire 1 mm in diamètre will sustain a weight of 4▿5 Cg, what weight might be sustained by a like wire 1 cm in diamètre?

5. What is the cube root of ▿091 125?

6. $(\sqrt[3]{16^6} \div \sqrt[3]{64}) - (4 \times \sqrt[3]{\triangledown 512}) = ?$

7. What will be the length of the edge of a cube that will have the same volume as a box 2▿5 m long, 40 dm wide, and ▿9261 m deep?

8. What is the 4th root of 5 636 405 776?

9. Extract the 7th root of 64 339 296 875.

10. Extract the 27th root of 134 217 728.

11. Find the 6th root of 308 915 776.

12. What is the length of the edge of a cube whose entire surface is 1014 ca?

13. If a round marble column 3 dm in diamètre contains 2▿5 s, what will be the diamètre of a similar column of equal length that contains $5\triangledown\dot{9}2\dot{5}$ s?

14. A pipe 9 cm in diamètre fills a cistern in 5 h. Find the diamètre of a pipe that, at the same pressure, would fill the same cistern in 1¼ h.

15. How much less will it cost to fence 25 Ha of land in the form of a square, than to fence it in the form of a rectangle whose breadth is but 25 Dm @ 51 c. per Dm?

16. What would it cost to fence 2450 a of land in the form of a right-angled triangle having a base just as long as its perpendicular at the rate of 5 c. per m?

17. What would be the surface of a globe 6 dm in diamètre?

18. What is the volume of a sphere whose radius is 5 Dm?

19. The surface of a certain sphere is the same as that of a cube whose edge is 1 Hm. Find,

a. The volume of the sphere;
b. The volume of the cube.

20. Find the cost of a stick of timber ▿4 m square and 18 m long @ 3 c. per litre.

21. Extract the square root of 195 364.

22. Required the square root of ▿0676.

23. Find the square root of 984 064.

24. Extract the square root of 5.

25. Extract the square root of 60▿487 129.

26. What is the square root of ▿0 000 012 321?

27. $\sqrt{79\,792\,266\,297\,612\,001}=?$

28. What is the square root of $\frac{216}{486}$?

29. What is the square root of $5\frac{1}{7}$?

30. Find the square root of $12\frac{24}{25}$.

31. Extract the square root of 4 356.

32. Find the square root of 186 624.

33. Extract the square root of 7225.

34. Extract the square root of 58 564.

35. Extract the square root of 9 604.

36. Find the square root of 13 225.

37. What is the square root of 11 881?

38. Find the square root of 994 009.

39. Required the square root of 29 855 296.

40. Find the square root of 2 050 624.

41. Extract the square root of $\frac{110}{121}$.

42. Find the square root of $\frac{147}{196}$.

43. Required the square root of $\frac{625}{6561}$.

44. Find the square root of $\frac{576}{18225}$.

45. What is the square root of ▿001 225?

46. What is the square root of 196▿2 801?

47. What is the square root of 2▿2 515 302 601?

48. Extract the square root of 58▿140 625.

49. What is the square root of 17▿75?

50. What is the square root of 9▿0 000 600 001?

51. Find the square root of 3 486 784 401.

52. What is the square root of 10 691▿56?

53. $\sqrt{1369} + \sqrt{1296} = ?$

54. What is the length of one side of a square whose area is 729 ca?

55. A square field contains 1 016 064 a. What is the length of one of its sides?

56. A square piece of land contains 361 Ha. Find the length of one of its sides.

57. If 250▿73 139 025 Ha of land be laid out in a square, how long will one side of it be?

58. What would be the length of one side of a square containing 21 170▿25 ca?

59. If the hypotenuse of a right-angled triangle is 12 m long, and the base 1 Dm, how long is the perpendicular?

60. If the foot of a ladder be placed 2 m from the side of a house, how long must the ladder be to reach to the eaves 4▿6 m from the ground?

61. Suppose a ladder 2 Dm long be placed 6 m from the foot of a tree, how far up the tree will it reach?

62. 2 persons start from the same point and go, the one due north 5 Mm, the other due west 88 Cm. Now how for apart are they, the globe's rotundity being disregarded?

63. How large a square stick of timber can be hewn out of a log 2▿ dm in diamètre at the smaller end?

64. Andrew Nelson has a ladder 36 m long, which, when set on the outer edge of a ditch 1▿8 Dm wide, just reaches the top of a wall on the other edge. What is the height of the wall?

65. A pole 3 Dm long is placed against a wall 17 m high and just reaches the top; it is then turned over and touches the opposite wall 25 m from the ground. How wide is the street?

66. If the area of a circle be 1764 ca, how many dm long will the side of a square of equal area be?

67. Emilio Aguinaldo has an army of 141 376 men. How many can he place in a row to form them into a square?

68. What is the distance through the opposite corners of an a?

69. The distance between the lower ends of 2 equal and opposite rafters is 3▿2 m, and the ridge of the roof is 12 dm higher up. How long is a rafter?

70. What distance apart are the opposite corners of a s?

71. What length of rope must Christian Dewit tie to his horse in order that he may feed over a Ha?

72. Extract the cube root of 328 509.

73. Find the cube root of 62 712 728 317.

74. What is the cube root of 1 953 125?

75. What is the cube root of 1 076 890 625?

76. What is the cube root of ▿697 864 103?

77. What is the cube root of 102 503▿232?

78. Find the cube root of 179 597▿069 288.

79. $\sqrt[3]{483\triangledown 736\,625}=?$

80. Extract the cube root of ▿636 056.

81. Extract the cube root of $\frac{8}{125}$.

82. Find the cube root of $\frac{3}{17}$.

83. Extract the cube root of $28\frac{3}{4}$.

84. Find the cube root of $32\frac{96}{125}$.

85. What is the cube root of 77 854v483?

86. Find the cube root of 405 224.

87. What is the cube root of 12 812 904?

88. Required the cube root of 15 625.

89. What is the cube root of 166 375?

90. Extract the cube root of 1 030 301.

91. Find the cube root of 4 492 125.

92. Find the cube root of 40 814 179v307.

93. Extract the cube root of $\frac{729}{1331}$.

94. What is the cube root of 12v812 904?

95. What is the cube root of v000 529 475 129?

96. $\sqrt[3]{\frac{297}{3375}\times\frac{125}{1375}}=?$

97. $\sqrt[3]{\text{v}4096-\text{v}236\,791\,307}=?$

98. What is the length of the edge of a cubical bin that will hold just as much as one 16 m long, 8 m wide, and 4 m deep?

99. What is the length of the edge of a cube containing 46 656 ml?

100. How many square dm in the surface of a cube whose volume is 91v125 s?

101. What is the length of the inner edge of a cubical bin that will hold 166v375 s?

102. Required the length of the edge of a cube whose volume is 6v4 Ms.

103. If a cannon ball 3 dm in diamètre will weigh 16 Mg, what will one 4 dm in diamètre weigh?

104. If a sphere of gold 2 cm in diamètre be worth $1200, what will one 7 cm in diamètre be worth?

105. If Frank Pettigrew, who is proportioned as was Goliath, and stands 1v75 m in height, weighs 91 Cg, what must Goliath have weighed, whose height was 3v3 m?

106. M. Bryan has a cube of clay whose edge is 9 173 dm long. He wishes to take out of it 5 cubes whose edges are to be 4v5 m, 62 dm, 3 m, 8 m, and 2 Dm long respectively. Required the length of the edge of the cube that can be formed out of the remaining clay.

107. Find the 8th root of 1 099 511 627 776.

108. What is the 9th root of 2v357 947 691?

109. Extract the 6th root of 4 826 809.

110. Find the 18th root of 387 420 489.

111. What is the 4th root of 19 987 173 376?

112. What is the volume of a cube whose entire surface is 1014 ca?

113. If a s of iron were formed into a bar a cm square, how long would the bar be?

114. It is desired to fence 282v27 Ha of land in the form of a rectangle 3 times as long as wide. How long will the field be?

115. A goat is fastened to the top of a post 2½ m high by a rope 2 Dm long. Over how much ground can he graze?

LONGITUDE AND TIME.

197. Since there are 400^{g} in a c., and since there are 24 h. in a d. and 60 m. in an h. and 60 s. in a m., and since the earth revolves once a day upon its axis, it will needs revolve $\frac{1}{24}$ of 400^{g} in an h., $\frac{1}{60}$ part of that in a m., and $\frac{1}{60}$ part of *that* again in a s.

It is to be hoped that the world will soon adopt an exclusively decimal mode of subdividing the day also.

$\frac{1}{24}$ of $400=\frac{1}{{}_{3}\cancel{24}}\times\frac{4\cancel{0}\cancel{0}^{50}}{1},=\frac{50}{3},=16\nabla 6$, making $16\nabla\dot{6}^{g}$, namely.

$16\nabla\dot{6}^{g}=16^{g}\cdot\cdot\,66^{\backprime}\cdot\cdot\,66\nabla\dot{6}^{\backprime\backprime}$.

$\frac{1}{60}$ of $16\nabla\dot{6}=\frac{1}{6\cancel{0}}\times\frac{5\cancel{0}}{3},=\frac{5}{18},=\nabla 2\dot{7}$, making $\nabla 2\dot{7}^{g}$.

$\nabla 2\dot{7}\times 100=27\nabla\dot{7}$.

$\nabla\dot{7}\times 100=77\nabla\dot{7}$.

$\frac{1}{60}$ of $16\nabla\dot{6}^{g}$ is therefore equivalent to $27^{\backprime}\cdot\cdot\,77\nabla\dot{7}^{\backprime\backprime}$.

$\frac{1}{60}$ of $\frac{1}{60}$ of $16\nabla\dot{6}^{g}=\frac{1}{{}_{12}\cancel{6}\cancel{0}}\times\frac{\cancel{5}^{1}}{18},=\frac{1}{216}^{g},=\nabla 004\,\dot{6}2\dot{9}^{g}$.

$\nabla 004\,\dot{6}2\dot{9}\times 100=\nabla 4\dot{6}2\dot{9}$.

$\nabla 4\dot{6}2\dot{9}\times 100=46\nabla\dot{2}9\dot{6}$.

Wherefore $\frac{1}{60}$ of $\frac{1}{60}$ of $\frac{1}{24}$ of $400^{g}=46\frac{8}{27}^{\backprime\backprime}$.

So the earth revolves $16\frac{2}{3}^{g}$, i. e., 16^{g}, $66^{\backprime}$, and $66\nabla\dot{6}^{\backprime\backprime}$ in an h.; $27\frac{7}{9}^{\backprime}$, i. e., $27^{\backprime}$ and $77\nabla\dot{7}^{\backprime\backprime}$ in a m.; and $46\frac{8}{27}^{\backprime\backprime}$, i. e., $46\nabla\dot{2}9\dot{6}^{\backprime\backprime}$ in a s.

a. The difference in longitude between 2 places named being given, the difference in time will be found by dividing the difference in longitude by $16\frac{2}{3}$ if it is given in ^{g}, and the quotient will be h.; by $27\frac{7}{9}$ if it is given in $^{\backprime}$, and the quotient will be m.; by $46\frac{8}{27}$ if it is given in $^{\backprime\backprime}$, and then the quotient will be s.

β. This difference, when found, is added to the time of the point west to give the time of the point east; but subtracted from the time of the point east to give the time of the point west.

γ. To find the difference in longitude between 2 designated points, the difference in time being given or indicated, *multiply* the given difference by $16\frac{2}{3}$ if it is given in h., and the product or answer will be ^{g}; by $27\frac{7}{9}$ if it is given in m., and the product will be $'$; by $46\frac{8}{27}$ if it is given in s., and then the product will be $''$.

δ. This difference, when found, must now be added to the longitude of the point east to give that of the point west, but subtracted from the longitude of the point west to give us that of the point east for *west* longitude, and vice versa for east longitude.

ϵ. If the difference is given in 2 or in all 3 denominations whether of time or of longitude, it will first be reduced to *one* denomination.

FINIS.

EPILOGUE.

198. The *author* of this little work, with many others, feels satisfied that certain little improvements in the mode of reckoning, as well as some slight changes in the nomenclature employed are needed. The following remarks and suggestions are, therefore, made with the modest hope that they may, peradventure, be looked upon as calculated to point to some extent in the direction of such reforms.

10 will, no doubt, always remain the scale on which men will count, even although it does contain but 2 prime factors, as 30, the lowest number containing 3, would be too inconveniently high.

The earth's meridian, like every other mundane thing, has proven a rather uncertain quantity from which to deduce a standard measure, and it may be questioned whether even the equator or the equatorial diamètre terminating at sea-level in the first and last meridian would have proven any more certain or stable.

Had it been fixed on a certain given round number of some given kind of light-waves, a number expressed by some power of 10, it would, in one sense, probably have been better. The present cm is a little more than 20000 blue-light wave lengths in length. "Science," such as it is, *now* says these given wave-lengths are fixed, uniform, and stable. But for how long will it continue to say so? A m of 10000 or of 100000 blue light-wave lengths *might* have had a *fixed* basis in nature, but how would we then deduce the g and the c from it, or refer them to it?

If the island of Elba, historic Elba, were now again erected into a sovereign, independent state, its inviolable independence forever guaranteed by all the other governments on earth, an observatory erected thereon well east-ward, and the meridian of that observatory accepted by *all* as the world's "first meridian," it would be well indeed. Its complement would pass through Behring Strait clear of both continents, and would be *the* proper date-line *throughout*. At all events, the complement of the world's "first meridian" *ought* to pass through Behring Strait clear of both continents and be the world's date-line throughout. It could probably be so located that it would cut through but *one* island, St. Lawrence, or Wmnak.

The length of the mètre has now become so very exactly fixed and determined that uncertainty in the matter can no longer be said to exist. Its name, too, and the prefixes used to multiply or divide it could not be materially improved upon as far as they go. Science has, however, now reached that stage where it has become imperatively necessary that they go further both ways. Nevertheless it would not be advisable to further subdivide the primary units. But, what is now called a mètre had better be called a Hektomètre. What we now call a centimètre could then be styled mètre. This would give us a millimètre only $\frac{1}{100}$ part of our present mm, and therefore so much more convenient for minute measurements. It would be only about twice the diamètre of the finest silk web. This would, moreover, contribute to greater simplicity and uniformity throughout; as, see below.

2 more Greek prefixes, the one indicative of 100 000, the other, of 1 000 000, could be introduced and *all* the (primary) units run out, (multiplied) thus far. This would then disembarrass us of those 2 ill-fitting words Quintal and Tonneau. For this purpose the terminal syllables of *δέκα* and *ἥκατον*, (10 and 100) could be prefixed to the initial syllable of *μύριοι*, (10 000) to make 2 words or prefixes etymologically meaning ten times ten thousand, (i.e., 100 000), and one hundred times ten thousand,

(i.e., 1 000 000). This would give us Kamy and Tomy. But this Tomymètre would then be only as long as our present Myriamètre.

In *some*, at least, of the measures, if not also in the case of the mètre itself, the primary units could, to very great advantage, be run out, (multiplied) considerably further. In fact, if they were all made to commence as small as they ought to, this would, in more than one case, become really necessary. 10 Tomymètre could, therefore, be called a millidimètre and the scale again run over to Tomydimètre, giving a very convenient measure and denominations, (for astronomers, etc., for instance). The prefix "di" from δίς, (twice), would seem to the author, after due deliberation, preferable to, and more practicable than the possibly, though hardly, more descriptive mega, me, or ga from μέγας.

In order to preserve more intact the cosmopolitan nature of the system, the primary units must never be pluralized. It can never become really necessary anyhow.

The sign or contraction for a subdivision or *sub*multiple of a primary unit should both times be the initial letter of the proper prefix written *small* and without the period. The sign of the *multiples* should be the initial letter of the proper prefix written *capital* and without the period. That of the primary units themselves should be their respective initials written small the first time, but capital whenever di is to be understood as having been prefixed, and always in both cases without the period, and, of course, without any attempt at pluralization. This would obviate all possible confusion. None of the contractions, (initials) could then ever be misunderstood.

It is most assuredly desirable that the names of *all* the primary units should be short, and, for uniformity's sake, of Greek origin. They should likewise all mean what they are intended to indicate; and they should, moreover, all commence with a consonant, which consonant should never be the same for any 2 of them.

It is therefore humbly and respectfully suggested,

α′. That the primary unit for length be the mètre, and be the ▿01 part of the mètre now established and in use;

β′. That the primary unit for surface extension be called a plat, and be a square mètre, (i. e., what is now called a square centimètre);

γ′. That the primary unit for weight be called a bar, and be the weight of a stère, (now ml), of pure water at its greatest density weighed in vacuum at the level of the sea at parallel 5 Tg, (now 50^{g}), i. e., *just half-way between the equator and a pole*, (hence what is now called a gramme—except as to the *latitude* of the place of weighing);

δ′. That the primary unit for value be called a ktes, and be a bar, (say very, *very* nearly what is now called a gramme) of pure gold or its value;

ε′. That the primary unit for electricity (or force,—power) be called a lekt, and be the (electric) force required to raise a bar, (now gramme) 1 m, (now centimètre) in 1 c in vacuum at sea-level on parallel 5 Tg;

ϛ′. That the primary unit for heat be called a therm, and be the increase of temperature required to expand the volume of mercury in vacuum ▿000 000 1 part of itself;

ζ′. That the primary unit for light be called a fos, and be the strength of light 1 lekt would give;

η′. That the primary unit for time be called a chron, and be the ▿000 000 1 part of a day, or while the earth is rotating say 4 g, i. e., 4 Hm, (4 of what is *now* called a mètre), at the equator;

θ′. That the primary unit for an angle or subdivision of a circle be called a gon, and be the ▿000 000 1 part of a right angle, (or a Hm, i. e., what we now call a mètre, on a great circle of the earth);

ι'. That the primary unit for volume be (in *every* case) called a stère, and be a cubic mètre, (what we now call a millilitre).

Thus all these units would also be derived directly or indirectly from the mètre, or at least referable thereto.

Every intelligent person will admit,

α'. That the present Mm is too short for the convenient reckoning of some measurements and the mm too long for some others;

β'. That are, commencing as it does with a vowel, and not being Greek either, is but ill chosen as a name for the unit of surface measure;

γ'. That the Ma is too small for some, and the ma far too large for many other practical purposes of measurement;

δ'. That there can be no plausible reason or even excuse offered for having 2 "primary units" of volume;

ε'. That the terms litre and gramme as used are to some extent misnomers;

ς'. That the ml is for many purposes far too big;

ζ'. That the franc is too small;

η'. That dollar is altogether too uncertain and indefinite, has had too many meanings;

θ'. That, for the sake of system and uniformity, *all* the primary units ought, like mètre, to be of Greek origin;

ι'. That our present time table is unnecessarily clumsy and irrational;

ια'. That our way of measuring heat, ay, and light, is no better, whilst we are attempting to measure power even still more awkwardly.

Mètre, plat, stère, and bar, it would seem to the author, are in every respect the 4 best and most suitable words or names that could be chosen for the purposes respectively for which they are here respectfully proposed, and he feels almost the same as to therm, fos, and chron.

Ktes, from κτῆσις, gain, ownership, earnings, or *acquisition*, may not be the best word that could be found to express the unit of value. In fact, the author would himself much prefer the word rubl, *if* it could be traced to Greek origin, which, however, he has not succeeded in doing. Tal, (from ταλάντον), chrem, (from χρήμα), and litre (from λίτρα) are all 3 objectionable on account of their initials—though litre *could* be used, (to indicate *value*, namely, the unit of money), if dyn, (from δύναμις), were used for electricity instead of lekt. Hyp, (from ὑπάρξις), or nom, (from νόμισμα) might answer. Or a word preferable to any here mentioned, and still meeting all the requirements hereinabove set forth, might, peradventure, be found.

It also readily and cheerfully conceded that it *may* possibly enough happen that a term more suitable than gon can be found for the primary unit in subdividing a right-angle or circle—though the fact that a word is already so generally understood should here, too, be given due weight. Thus the words rhyt, (from ῥυτίς, a wrinkle), kom, (from κόμμα, a segment), tref, (from τρεφείν, to turn), and gyr, (from γύρος, a circle), and some others have been considered and, at least for the present, postponed to gon.

It is hereby respectfully suggested that there be kept with the world's proto- or standard-Hektomètre and chilobar a standard thermomètre also. This thermomètre should be made, if possible, of some *simple* body the least affected, (expanded or contracted) by change of temperature, and through which the mercury within could be seen, (or photographed)—by the aid of the X ray, or in *some* way. Then the walls of this thermomètre, as well of the bulb as of the tube, should, or course, have a uniform, fixed thickness throughout. Would platinum or iridium do? The bulb could be made to hold just exactly 1 Ds, (what we are *now* calling a centilitre), and the bore of the tube could be exactly of the capacity of a cp, (now known as a square millimètre). Then the point just 1 Hm, (what we now call a mètre), above the bulb could be marked 1 Tt, thus making a t swell or expand the mercury v000 000 1 part of its own volume. In such a thermomètre an increase of 1 t in temperature would raise the mercury $\frac{1}{10}$ of 1 mm, (i. e., what would *now* be termed $\frac{1}{1000}$ part of a mm). The congealing point of mercury had best be called zero, and just enough mercury put into this protothermomètre to exactly fill the *bulb* at this congealing point. The proportions, etc., here proposed may perhaps not be the best, but a prototype thermomètre somewhat after this fashion and based on the mètre seems to be needed. To have but one point from which to reckon and determine the temperature instead of 2 as now, would already be in itself a most decided advantage, especially so where that one point is certainly more constant and definite than either of the 2 now used. The readings would be + and — from such zero.

The readings of all of these, as well as of the gon, chron, lekt, and fos would very readily, easily, soon, naturally, and smoothly adjust themselves after a little practice.

What we now call the 25th of March must appear to every Christian, and even to others to be the most appropriate day on which to commence a New Year, inasmuch as the vernal equinox is not always on the same date anyhow. 2 of the names, (say pisces and libra) could be dropped out of the list of names applied to the star-groups in the (redistricted) zodiac, and 2 of the names applied to months,—say January and July, could also be so omitted. Then the first month in the year, April, could have 37 days; the next month, May, 36 days; the next, June, 37 days; the next month, August, 36 days; and so on alternately throughout, but giving December, the last month of the year, 37 days in leap-year. And might not the Greek ordinals, Protos, Deuteros, Tritos, Tetartos, Pemptos, Hektos, Hebdomos, Ogdoos, Enatos, and Dekatos be used as names for the months, rendering these names cosmopolitan also? If no 2 of these commenced with the same letter, it would be better again. Or peradventure 10 names could be selected of Greek origin and commencing with the first 10 letters of the alphabet in correct succession. But before we have a universal alphabet, such an arrangement would avail but little. Some day, when the number of *living* languages will have become vastly reduced, the world may expect to see one entirely comprehensive alphabet gotten up with a separate character for each and every separate and distinct elementary sound used in all or any of these languages, from which alphabet, as from a public fountain, each nation will select for *its* alphabet the particular characters representing the sounds and all of the sounds that *it* uses in *its* speech. Then no letter or character will any longer mean one thing, represent one sound, in one country and another elementary sound in another country or place.

It could easily be arranged by legislative enactments or otherwise, so, that all debts due and unpaid, (or at least all of a class), would draw a fixed, (uniform) rate of interest, if interest we *must* have, to be computed in even months regardless of odd days, much as it now is figured on delinquent taxes. For instance, a debt, not paid on the day upon which it becomes due, or on the then next business day following, might be made to at once bear one full month's interest, adding another full month's interest, for every (calendar) month into which it may be permitted to run beyond the first business day thereof. This where the rate is low; where high, the debtor might be afforded the *whole* month in which to make payment.

There being forever diversity of tongues upon this mundane sphere, uniformity in the *pronunciation* of the names of these primary units or of the prefixes can never be fully attained throughout and need not be expected, though certainly *desirable* enough. But uniformity in SPELLING them can very easily be practiced the world over, and must, or too much is lost. Uniformity in spelling is far, far more important here than uniformity in pronunciation. Uniformity in spelling these names and prefixes is, in fact, simply and imperatively necessary and essential. *These primary units, therefore,* (let it here be reiterated and emphasized), *ought never to be pluralized.* Neither should a period be written with (after) their signs or initials, nor after those of the prefixes used, whether small or capital. This, besides making it easier to employ them, will also render them more readily distinguishable from the same letters when they are being used as abbreviations, signs, or initials of other words.

The author has often heard the very just and reasonable claim made that the "decimal point" ought to be a mark more conspicuous than it is, and so more readily distinguishable. Its name, too, is long. It is here suggested, therefore, that the Greek capital D, delta, "Δ" be employed instead, but inverted as is done in this work; and that its altitude be uniformly half that of the other figures with which used. Δ is the initial letter of the word δήκεμ or δέκα and, leaving out the adjective "inverted," it could, to advantage, still be spoken of as simply the *delta.* Or would some other letter of the Greek alphabet prove preferable? γ or ρ, for instance? ρ might be the best. Or would it not be even better still to use no distinctive mark at all to denote the decimal place, but, in lieu thereof, encircle the figure in the units' place, thus: 24⑦, 32⑤61, ⓪003, ⓪4, i.e., four tenths? or draw a straight line through it, thus: 247, 32561, 0003, 04? This, though a little less convenient, would, perhaps, be more readily and easily read, since it is *really* from the *units'* place rather than from the decimal point that we proceed.

The other figures, the cipher and the 9 digits ought certainly to be written of even, uniform altitude and on the line, the rather fanciful but impractical dreamings of certain parties to the contrary notwithstanding.

Where a considerable number of deltas are to be placed in column, a perpendicular line, beginning at the top in an inverted delta can be used, Thus ▽.

199. To repeat, then, and to tabulate we would have,
milli, (from mille, 1000), the ▽001 part;
centi, (from centum, 100), the ▽01 part;
deci, (from decem, 10), the ▽1 part;

Deka, (from δέκα, 10), 10, (or ten times);
Hekto, (from ἑκατόν, 100), 100, (times);
Chilo, (from χίλιοι, 1 000), 1 000;
Myria, (from μύριοι, 10 000), 10 000;
Kamy, (from δεκακιςμύριοι, 100 000), 100 000;
Tomy, (from ἑκατοντακιςμύριοι, 1 000 000), 1 000 000;
di, from δίς, twice), the second or greater;

mètre, (from μέτρον, a measure), about the ▽000 000 001 part of the distance on the earth's face from the equator to a pole, either pole;

plat, (from πλάτυς, flat), a square m;

stère, (from στέρεος, solid), a cubic m;

bar, (from βάρος, weight), the weight of a s of pure water at its greatest density weighed in vacuum at sea-level just midway between the equator and a pole, i.e., at 5 Tg from the equator north or south;

ktes, (from κτῆσις, earnings, acquisition), a b of gold or its value;

lekt, (from ἤλεκτρον, amber), the (electric) force required to raise a b 1 m in 1 c in vacuum at sea-level in latitude 5 Tg;

therm, (from θέρμη, heat), the amount of heat required to swell mercury v000 000 1 part of its own volume;

fos, (from φῶs, light), the light a l will give;

chron, (from χρόνος, time), the v000 000 1 part of a day or while the earth rotateth, say 4 g, i. e., at the equator, 4 Hm;

gon, (from γωνία, an angle), the v000 000 1 part of a right-angle, that is to say, about a Hm on the earth's surface.

The signs, initials, or abbreviations are:

m for milli;
c for centi;
d for deci;

D for Deka;
H for Hekto;
C for Chilo;
M for Myria;
K for Kamy;
T for Tomy;
m for mètre;
p for plat;

s for stère;
b for bar;
k for ktes;
l for lekt;
t for therm;
f for fos;
c for chron;
g for gon;
M for dimètre;
P for diplat;
S for distère;
B for dibar;

K for diktes;
L for dilekt;
T for ditherm;
F for difos;
∟ for right-angle;
c. for circle;
d for day;
w for week;
m. for month;
y for year;
D for decade; and
C for century.

"Odd" or "even" must be prefixed to the m., and "leap" or "common" to the y whenever necessary to make the distinction *as to the number of days therein.* But y is always taken to mean a common year unless preceded by "leap."

Of course C would at this rate also indicate a dichron and G a digon could they be used; but they could scarcely be given a practical intelligent meaning. The one would be equivalent to 1 000 days, i. e., to 2 y ·· 7 m. ·· 14 d; the other, to 250 times clean round the circle. Hence no . is needed after C to denote century. The use of the same initial for a primary unit and also at the same time for a prefix can occasion no difficulty, since the prefixes are never used *alone*.

200. Presenting now the following tables, the author hereby most respectfully craves permission to make his bow to a patient and indulgent and, he hopes and trusts, to an appreciative public, very cordially inviting all those who may kindly deign to criticise this work, to address themselves to himself directly so that he may graciously be accorded the honor and pleasure of profiting and learning from them.

L. S. C.

201. TABLE 1.

10 millimètre, (mm)=1 centimètre, (cm);
10 centimètre=1 decimètre, (dm);
10 decimètre=1 mètre, (m);
10 mètre=1 Dekamètre, (Dm);
10 Dekamètre=1 Hektomètre, (Hm);
10 Hektomètre=1 Chilomètre, (Cm);
10 Chilomètre=1 Myriamètre, (Mm);
10 Myriamètre=1 Kamymètre, (Km);
10 Kamymètre=1 Tomymètre, (Tm);
10 Tomymètre=1 millidimètre, (mM);
10 millidimètre=1 centidimètre (cM);
10 centidimètre=1 decidimètre, (dM);
10 decidimètre=1 dimètre, (M);
10 dimètre=1 Dekadimètre, (DM);
10 Dekadimètre=1 Hektodimètre, (HM);
10 Hektodimètre=1 Chilodimètre, (CM);
10 Chilodimètre=1 Myriadimètre, (MM);
10 Myriadimètre=1 Kamydimètre, (KM);
10 Kamydimètre=1 Tomydimètre, (TM).

Table 2.

10 milliplat, (mp)=1 centiplat, (cp);
10 centiplat=1 deciplat, (dp);
10 deciplat=1 plat, (p);
10 plat=1 Dekaplat, (Dp);
10 Dekaplat=1 Hektoplat, (Hp);
10 Hektoplat=1 Chiloplat, (Cp);
10 Chiloplat=1 Myriaplat, (Mp);
10 Myriaplat=1 Kamyplat, (Kp);
10 Kamyplat=1 Tomyplat, (Tp);
10 Tomyplat=1 millidiplat, (mP);
10 millidiplat=1 centidiplat, (cP);
10 centidiplat=1 decidiplat, (dP);
10 decidiplat=1 diplat, (P);
10 diplat=1 Dekadiplat, (DP);
10 Dekadiplat=1 Hektodiplat, (HP);
10 Hektodiplat=1 Chilodiplat, (CP);
10 Chilodiplat=1 Myriadiplat, (MP);
10 Myriadiplat=1 Kamydiplat, (KP);
10 Kamydiplat=1 Tomydiplat, (TP).

Table 3.

10 millistère, (ms) = 1 centistère, (cs);
10 centistère = 1 decistère, (ds);
10 decistère = 1 stère, (s);
10 stère = 1 Dekastère, (Ds);
10 Dekastère = 1 Hektostère, (Hs);
10 Hektostère = 1 Chilostère, (Cs);
10 Chilostère = 1 Myriastère, (Ms);
10 Myriastère = 1 Kamystère, (Ks);
10 Kamystère = 1 Tomystère, (Ts);
10 Tomystère = 1 millidistère, (mS);
10 millidistère = 1 centidistère, (cS);
10 centidistère = 1 decidistère, (dS);
10 decidistère = 1 distère, (S);
10 distère = 1 Dekadistère, (DS);
10 Dekadistère = 1 Hektodistère, (HS);
10 Hektodistère = 1 Chilodistère, (CS);
10 Chilodistère = 1 Myriadistère, (MS);
10 Myriadistère = 1 Kamydistère, (KS);
10 Kamydistère = 1 Tomydistère, (TS).

TABLE 4.

10 millibar, (mb)=1 centibar, (cb);
10 centibar=1 decibar, (db);
10 decibar=1 bar, (b);
10 bar=1 Dekabar, (Db);
10 Dekabar=1 Hektobar, (Hb);
10 Hektobar=1 Chilobar, (Cb);
10 Chilobar=1 Myriabar, (Mb);
10 Myriabar=1 Kamybar, (Kb);
10 Kamybar=1 Tomybar, (Tb);
10 Tomybar=1 millidibar, (mB);
10 millidibar=1 centidibar, (cB);
10 centidibar=1 decidibar, (dB);
10 decidibar=1 dibar, (B);
10 dibar=1 Dekadibar, (DB);
10 Dekadibar=1 Hektodibar, (HB);
10 Hektodibar=1 Chilodibar, (CB);
10 Chilodibar=1 Myriadibar, (MB);
10 Myriadibar=1 Kamydibar, (KB);
10 Kamydibar=1 Tomydibar, (TB).

TABLE 5.

10 milliktes, (mk) = 1 centiktes, (ck);
10 centiktes = 1 deciktes, (dk);
10 deciktes = 1 ktes, (k);
10 ktes = 1 Dekaktes, (Dk);
10 Dekaktes = 1 Hektoktes, (Hk);
10 Hektoktes = 1 Chiloktes, (Ck);
10 Chiloktes = 1 Myriaktes, (Mk);
10 Myriaktes = 1 Kamyktes, (Kk);
10 Kamyktes = 1 Tomyktes, (Tk);
10 Tomyktes = 1 millidiktes, (mK);
10 millidiktes = 1 centidiktes, (cK);
10 centidiktes = 1 decidiktes, (dK);
10 decidiktes = 1 diktes, (K);
10 diktes = 1 Dekadiktes, (DK);
10 Dekadiktes = 1 Hektodiktes, (HK);
10 Hektodiktes = 1 Chilodiktes, (CK);
10 Chilodiktes = 1 Myriadiktes, (MK);
10 Myriadiktes = 1 Kamydiktes, (KK);
10 Kamydiktes = 1 Tomydiktes, (TK).

TABLE 6.

10 millilekt, (ml) = 1 centilekt, (cl);
10 centilekt = 1 decilekt, (dl);
10 decilekt = 1 lekt, (l);
10 lekt = 1 Dekalekt, (Dl);
10 Dekalekt = 1 Hektolekt, (Hl);
10 Hektolekt = 1 Chilolekt, (Cl);
10 Chilolekt = 1 Myrialekt, (Ml);
10 Myrialekt = 1 Kamylekt, (Kl);
10 Kamylekt = 1 Tomylekt, (Tl);
10 Tomylekt = 1 millidilekt, (mL);
10 millidilekt = 1 centidilekt, (cL);
10 centidilekt = 1 decidilekt, (dL);
10 decidilekt = 1 dilekt, (L);
10 dilekt = 1 Dekadilekt, (DL);
10 Dekadilekt = 1 Hektodilekt, (HL);
10 Hektodilekt = 1 Chilodilekt, (CL);
10 Chilodilekt = 1 Myriadilekt, (ML);
10 Myriadilekt = 1 Kamydilekt, (KL);
10 Kamydilekt = 1 Tomydilekt, (TL).

TABLE 7.

10 millitherm, (mt)=1 centitherm, (ct);
10 centitherm=1 decitherm, (dt);
10 decitherm=1 therm, (t);
10 therm=1 Dekatherm, (Dt);
10 Dekatherm=1 Hektotherm, (Ht);
10 Hektotherm=1 Chilotherm, (Ct);
10 Chilotherm=1 Myriatherm, (Mt);
10 Myriatherm=1 Kamytherm, (Kt);
10 Kamytherm=1 Tomytherm, (Tt);
10 Tomytherm=1 milliditherm, (mT);
10 milliditherm=1 centiditherm, (cT);
10 centiditherm=1 deciditherm, (dT);
10 deciditherm=1 ditherm, (T);
10 ditherm=1 Dekaditherm, (DT);
10 Dekaditherm=1 Hektoditherm, (HT);
10 Hektoditherm=1 Chiloditherm, (CT);
10 Chiloditherm=1 Myriaditherm, (MT);
10 Myriaditherm=1 Kamyditherm, (KT);
10 Kamyditherm=1 Tomyditherm, (TT).

TABLE 8.

10 millifos, (mf)=1 centifos, (cf);
10 centifos=1 decifos, (df);
10 decifos=1 fos, (f);
10 fos=1 Dekafos, (Df);
10 Dekafos=1 Hektofos, (Hf);
10 Hektofos=1 Chilofos, (Cf);
10 Chilofos=1 Myriafos, (Mf);
10 Myriafos=1 Kamyfos, (Kf);
10 Kamyfos=1 Tomyfos, (Tf);
10 Tomyfos=1 millidifos, (mF);
10 millidifos=1 centidifos, (cF);
10 centidifos=1 decidifos, (dF);
10 decidifos=1 difos, (F);
10 difos=1 Dekadifos, (DF);
10 Dekadifos=1 Hektodifos, (HF);
10 Hektodifos=1 Chilodifos, (CF);
10 Chilodifos=1 Myriadifos, (MF);
10 Myriadifos=1 Kamydifos, (KF);
10 Kamydifos=1 Tomydifos, (TF).

TABLE 9.

10 millichron, (mc)=1 centichron, (cc);
10 centichron=1 decichron, (dc);
10 decichron=1 chron, (c);
10 chron=1 Dekachron, (Dc);
10 Dekachron=1 Hektochron, (Hc);
10 Hektochron=1 Chilochron, (Cc);
10 Chilochron=1 Myriachron, (Mc);
10 Myriachron=1 Kamychron, (Kc);
10 Kamychron=1 Tomychron, (Tc);
10 Tomychron=1 day, (d);
7 days=1 week, (w);
$5\frac{2}{7}$ weeks=1 odd month, (m.);
$5\frac{1}{7}$ weeks=1 even month, (m.);
10 months=1 year, (y);
10 years=1 decade, (D);
10 decades=1 century, (C).

Table 10.

10 milligon, (mg) = 1 centigon, (cg);
10 centigon = 1 decigon, (dg);
10 decigon = 1 gon, (g);
10 gon = 1 Dekagon, (Dg);
10 Dekagon = 1 Hektogon, (Hg);
10 Hektogon = 1 Chilogon, (Cg);
10 Chilogon = 1 Myriagon, (Mg);
10 Myriagon = 1 Kamygon, (Kg);
10 Kamygon = 1 Tomygon, (Tg);
10 Tomygon = 1 right-angle, (∟);
4 right-angles = 1 circle, (c.).

The fore part of tables 9 and 10 together with all of the other 8 tables last preceding can be better given amalgamated into one, as is done in Table 11 following.

Or the signs merely, the proper initials alone (with their explanations) would answer, as in Table 12.

Or these explanations could be omitted, as in Table 13.

Or again, they may be subjoined in notes below, or rather opposite, as is done with Table 14.

TABLE 11.

milli, (m);		plat, (p);
centi, (c);		stère, (s);
deci, (d);		bar, (b);
		mètre, (m);
Deka, (D);		ktes, (k);
Hekto, (H);		lekt, (l);
Chilo, (C);		therm, (t);
Myria, (M);		fos, (f);
Kamy, (K);	Scale 10.	gon, (g);
Tomy, (T);		chron, (c);
milli, (m);		plat, (P);
centi, (c);		stère, (S);
deci, (d);		bar, (B);
	di	mètre, (M);
Deka, (D);		ktes, (K);
Hekto, (H);		lekt, (L);
Chilo, (C);		therm, (T);
Myria, (M);		fos, (F).
Kamy, (K);		
Tomy, (T);		

TABLE 12.

m
c
d
D
H
C
M
K
T
m
c
d
D
H
C
M
K
T

Scale 10.

di

p = 1 square m;
s = 1 cubic m;
b = weight of 1 s of densest water in vacuum at sea-level 5 Tg;
m = say ▿0000000005 of the earth's meridian;
k = 1 b of gold or its value;
l = force to raise 1 b 1 m from sea-level in 1 c in vacuum 5 Tg;
t = heat to swell mercury ▿000 000 1 of its volume;
f = the light 1 l will give;
g = ▿000 000 1 of a ∟;
c = ▿000 000 1 of a d;
P = 10 000 Tp;
S = 10 000 Ts;
B = 10 000 Tb;
M = 10 000 Tm;
K = 10 000 Tk;
L = 10 000 Tl;
T = 10 000 Tt;
F = 10 000 Tf.

TABLE 13.

m		p
c		s
d		b
		m
D		k
H		l
C		t
M		f
K		g
T		c
m	Scale 10.	P
c		S
d		B
	di	M
D		K
H		L
C		T
M		F.
K		
T		

TABLE 14.

m		p
c		s
d		b
		m
D		k
H		l
C		t
M		f
K	Scale 10.	g
T		c
m		P
c		S
d		B
	di	M
D		K
H		L
C		T
M		F.
K		
T		

NOTES.

m=(about) ▿000 000 000 5 of the earth's meridian.
p=1 square m.
s=1 cubic m.
b=weight of a s of densest water at sea-level in vacuum at 5 Tg.
k=1 b of gold or its value.
l=force to raise 1 b 1 m from sea-level in 1 c in vacuum at 5 Tg.
t=heat to swell mercury ▿000 000 1 part of its volume.
f=the light 1 l will give.
g=▿000 000 1 of a right-angle.
c=▿000 000 1 of a day.
M=10 000 Tm.
P=10 000 Tp.
S=10 000 Ts.
B=10 000 Tb.
K=10 000 Tk.
L=10 000 Tl.
T=10 000 Tt.
F=10 000 Tf.

DEO GRATIAS.

ANSWERS.

To Point 35.

1. 607 002 251 778v16 000 014.
2. a. 18 190 276v14;
b. 13 732 394v64 218.
3. 984.
4. 494 Cm.
5. a. 727v354 Cg;
b. 770v964 Cg;
c. 796v554 Cg;
d. 840v164 Cg.
6. 29 958 140.
7. 2 576v45 Ha.
8. 197v718 a.
9. 1 662 Cm.
10. 800 Cm.
11. 1 081v208.
12. 13 354v1 082.
13. 11 547v35.
14. 44 000 094 007v0 998 755.
15. a. 298;
b. 82.
16. 34 192.
17. 55.
18. $314v3.
19. $303v31.
20. 1 073v47.
21. 3 926.
22. a. 2 302;
b. 1 477;
c. 3 779.
23. 203 070 041v77 530 015.
24. 2 092 a.
25. 629 715 213 818 326 008 225 591 547 396 449v803 200 003.
26. $542v12.
27. 4 197 108 774v778 955 432 151.
28. $1 188v95.
29. 2 001v555.
30. 15v762 Cg.

Point 41.

1. 776 Cm.
2. 144 Mm.
3. 1 648v25 s.
4. 128 517 g.
5. 573 000 Ma.
6. a. 12 645 700 Ma;
b. 13 218 700 Ma;
c. 3 434 900 Ma.
7. 220 150 300 a.
8. 170 940 900 Ma.
9. 2 746v9 m.
10. 87 Cg.
11. 5.
12. 34.
13. 5.
14. 185.
15. 12.

16. 795 499.
17. 134 273 808.
18. 14 016 831.
19. 41 413 441.
20. 3 229 411.
21. 75 467 711.
22. 8 279 979.
23. 25 896▿3.
24. 34▿5.
25. 6 512.
26. 5 735▿5.
27. 28 528▿7.
28. 733 265▿5.
29. 1 021 777.
30. 20▿606.
31. 67▿23.
32. 88▿11.
33. 561▿13.
34. 42▿7 101.
35. 35 133▿87.
36. 1 381 376▿6 629.
37. 791 999 989▿944.
38. 12 135 964 215.
39. 10 221▿314 049 588.
40. 1 150 301 038▿77 737.
41. 557▿12 131 415 162.
42. 4▿68 904 387 343.
43. 5▿80 031 306.
44. 89 147 061▿5.
45. 1 957 794 433▿14.
46. 706 068.
47. 5 105 304 441▿35 018 014.
48. 0, 2, 4, 6, 8, 10, 12, 14, 16, and 18.
49. ▿1, ▿2, ▿3, ▿4, ▿5, ▿6, ▿7, ▿8, and ▿9.
50. 1▿98, 1▿96, 1▿94, and 1▿92.
51. 46 735▿64.
52. 28▿217 888 882.
53. 853 441 474▿9.
54. 362 634 584 899 617▿35 674 845 673.
55. $221▿65.
56. $3 976▿12.
57. $3 000 000.
58. $1 488▿85.
59. g. $3 098▿18.
60. 1 002.
61. 862.
62. 90▿9.
63. 1▿78.
64. 65▿5.
65. 4 610.
66. 169 103▿165.
67. In 1784.
68. $193▿88.
69. g. $308▿85.
70. $227▿19.
71. a. 80 000 000;
b. 320 000 000.
72. 978 357.
73. a. 1 469 400 Ma;
b. 7 598 100 Ma;
c. 9 067 500 Ma;
d. 1 172 800 Ma;
e. 16 369 000 Ma;
f. 19 307 800 Ma;
g. 36 849 600 Ma;

h. 28 078 700 Ma.

74. $72 835v48.

Point 56.

1. 1 089 Hl.
2. 2 460 Cg.
3. 10 080.
4. 3 276.
5. 2 884.
6. 2 000 000.
7. 2 762 500.
8. 200 000 000.
9. 40 000 000.
10. 510 000.
11. $340.
12. $375.
13. $186v9.
14. $10 000.
15. 78 960.
16. 6 195v6.
17. 42 100.
18. 532v124.
19. 26 052 600.
20. 1 342 000.
21. 4 000 000.
22. v2 562 509 502.
23. v0 018.
24. 47 615 177v37 978.
25. 78 027.
26. 36v27.
27. 12 024.
28. v3 627.
29. 2 967v0 975.
30. 37v2 105.
31. 101v202.
32. 676 689 606v12.
33. 0.
34. 169v935 528.
35. v001.
36. v1 038 996.
37. 915v4 425.
38. 28v09 477.
39. 5 066 509v6.
40. 200v002.
41. 448 452 480.
42. 4 951 328 536 245.
43. 943 921 943 010v909 368.
44. 134 689.
45. a. $8 422v65;
b. 110 100;
c. 280 755 Hg.
46. 34 195 068 775 816 971v84.
47. 93 619v534 251 032 064.
48. 163 567 487 189 495 070 000.
49. 32v7 170 528 203 833 506 052.
50. 896v280 275 315 742.
51. 73 106 504 915 926 214v9 957 625.
52. 0.
53. 9 346 775 811 749 230v6 362.
54. 0.
55. 10 267 499 520 000.
56. 31 711 801 208 027v8 801 392.
57. 168.
58. 362 880.
59. 199 491.

60. 364
61. 30 240.
62. 744.
63. 672.
64. 43 200.
65. 3 000 000.
66. 173 200 000.
67. 200.
68. 1 000 000.
69. 625▿1.
70. 254 000.
71. 2 560 000.
72. 101 000.

Point 65.

1. 12 583▿3̇.
2. 3 750 000 d.
3. 2 288 940 000.
4. 4▿16 Hg.
5. 6 768.
6. $5 000.
7. 4 057 826 700 000▿1.
8. 1 555▿556.
9. 39▿6̇82 539̇ wk.
10. 1▿3̇ Q.
11. 1 205▿93.
12. 2 511.
13. 17▿76.
14. 328.
15. 1 232 741▿640 625.
16. 1 282▿6 605.
17. 14 020 019 658▿036 262+.
18. ▿0 375.
19. 100 000.
20. $19 108▿9̇ 108̇.
21. 5▿836̇.
22. 6 678 666 777 873 056 558455▿497 501+.
23. 15▿605 729+.
24. 3 255▿2 083̇ d.
25. 1▿1.
26. 9.
27. 78 994 474 005 349▿608.
28. 30▿9.
29. $19▿89.
30. 6 121 493▿2 340 826.
31. 6 250 694▿4 583̇.
32. 27 439▿0 015.
33. 781▿4 687 256 Mm.
34. 1 062▿9 557 5̇39 682̇ wk.
35. 3 854▿7̇00 854̇.
36. 2▿952 879+.
37. 6 816▿69.
38. 29▿92 166 Ha.
39. a. $15 446▿05 055,
b. $1▿757.
40. 21 y.
41. 36 525.
42. 52 594 560.
43. $91▿25.
44. 10▿456 295+.
45. 7 T.
46. $187▿7.
47. 45.
48. 3▿53 878 T.
49. 3▿045 004+ c.
50. 9▿2̇85 714̇ ds.

51. ▿122 977+.
52. 5 000 000.
53. $21 063▿6̇.
54. 83 456 000.
55. 86 704▿8.
56. 22 433▿468 204+ m.
57. 8▿875.
58. 9▿5.
59. 2 400.
60. 19 Cg.
61. 350▿855 192+.
62. 4▿926.
63. $60.
64. 26 410.
65. ▿1̇12 554̇.
66. 91 612▿125.
67. 5 631▿815 826+.
68. 82 610▿491 532+.
69. 64.
70. 8▿808 011+ ms.
71. 18▿97 262 2̇70̇.
72. 16▿6̇.
73. $8▿05.
74. 21▿3̇04 029̇.
75. $2 974▿7.
76. 1▿4 761.
77. $701 896 300▿13.
78. 4 388 157▿894 736+ mg.
79. $3 669▿74.
80. 9 200 000.
81. ▿9.
82. $4▿955.
83. 1 261 232 856.

Point 70.

1. 10.
2. 17.
3. 1.
4. 28.
5. 34 560.
6. 40.

Point 74.

1. 2 520.
2. 3 780.
3. 147 400.
4. 11 880.
5. 2 520.
6. 98 280.
7. 60 480.
8. 6 985 440.
9. 34 320.
10. 240.
11. 33 600.
12. 6 300.
13. 55 440.
14. 8 640.
15. 10 353 000.
16. 990 990.
17. 159 603 444.
18. 7 163 200.
19. 1 204 827 624.

Point 79.

1. 69▿6 345 2̇23 013̇.
2. 99▿60 313 5̇74̇.
3. 172▿2 753 607 8̇20 176 612 255̇.

4. 8 872▿888 845̇ 559̇.
5. ▿899 902 0̇76 678 964 987-848̇.
6. ▿103 225 841 354̇.
7. 107▿041̇ 050 074 137̇.
8. 1▿20 266 103 6̇61 759 869-680̇.
9. 92▿32 222 228 374̇ 516 172-434̇.
10. ▿6 545 454 555 35̇4̇.
11. 2 319▿131 8̇90 152 078 270̇.
12. 8▿5 152 542 023̇ 475̇.
13. 112▿04̇6̇.
14. 49▿500 008̇ 305̇.

Point 84.

1. $\frac{672}{100}$.
2. $\frac{34}{1}$.
3. $\frac{83}{1}$.
4. $\frac{38015}{10000}$.
5. $\frac{623}{10}$.
6. $\frac{12005}{10000}$.
7. $\frac{59098}{10000}$.
8. $\frac{871}{10}$.
9. $\frac{781}{10}$.
10. $\frac{4}{1000}$.
11. $\frac{1004}{100000}$.
12. ▿1.
13. ▿7.
14. ▿75.
15. ▿75.
16. 18.
17. 5▿3.
18. 1▿1.
19. 1▿08.
20. 205▿7.

Point 90.

1. $\frac{61}{99}$.
2. $\frac{163}{333}$.
3. $\frac{808}{999}$.
4. $\frac{40}{99999}$.
5. $\frac{32}{9999}$.
6. $\frac{1}{99}$.
7. $\frac{4}{9}$.
8. $\frac{494}{999}$.
9. $\frac{1099}{9999}$.
10. $\frac{400}{33}$.
11. $\frac{896}{2475}$.
12. $\frac{3599}{4995}$.
13. $\frac{2729}{18000}$.
14. $\frac{10}{11}$.
15. $\frac{70}{999}$.
16. $\frac{2943}{11110}$.
17. $\frac{50}{9}$.
18. $\frac{1982}{99}$.
19. $\frac{100}{9}$.
20. $\frac{888888881}{900000000}$.
21. $\frac{1}{10}$.
22. $\frac{301}{396}$.
23. $\frac{276}{1}$.
24. $\frac{282}{5}$.
25. $\frac{1}{1}$.
26. $\frac{123}{1}$.
27. $\frac{701821}{990}$.

Point 93.

1. $\frac{839}{11}$.
2. $\frac{280}{19}$.
3. $\frac{1125}{2}$.
4. $\frac{263}{12}$.
5. $\frac{15}{14}$.
6. $\frac{1680}{19}$.
7. $\frac{5289}{10}$.
8. $\frac{821791}{13}$.
9. $\frac{3}{2}$.
10. $\frac{175}{8}$.
11. $\frac{251}{5}$.
12. $\frac{171}{4}$.
13. $\frac{194}{11}$.
14. $\frac{327}{11}$.
15. $\frac{41}{5}$.
16. $\frac{85}{9}$.
17. $\frac{2819}{5}$.
18. $\frac{3235}{4}$.
19. $\frac{8491}{6}$.
20. $\frac{509}{2}$.
21. $\frac{4603}{8}$.
22. $\frac{5937}{7}$.
23. $\frac{8744}{9}$.
24. $\frac{262}{3}$.
25. $\frac{5457}{2}$.
26. $\frac{16615}{11}$.
27. $\frac{7088}{13}$.
28. $\frac{1019}{5}$.
29. $\frac{2263}{484}$.
30. $\frac{9303}{101}$.
31. $\frac{13328}{15}$.
32. $\frac{1652}{17}$.
33. $\frac{446485}{712}$.
34. $\frac{19177}{25}$.
35. $\frac{47998}{71}$.

Point 96.

1. $3\frac{4}{9}$.
2. $141\frac{1}{5}$.
3. 41.
4. $6178\frac{4}{10}$.
5. $3\frac{69}{88}$.
6. 33.
7. $17\frac{2}{11}$.
8. $17\frac{5}{8}$.
9. 203.
10. $7\frac{21}{50}$.

Point 98.

1. $\frac{11}{7}$.
2. $\frac{131}{1250}$.
3. $\frac{976}{5}$.
4. $\frac{1}{143}$.
5. $\frac{1}{2}$.
6. $\frac{63}{64}$.
7. $\frac{8}{125}$.
8. $\frac{3}{13}$.
9. $\frac{11111}{2002}$.
10. $\frac{23321}{11321}$.
11. $\frac{9}{50}$.
12. $\frac{5}{111}$.
13. $\frac{5}{6}$.
14. $\frac{10}{91}$.
15. $\frac{7}{17}$.
16. $\frac{3262}{2827}$.
17. $\frac{888}{8773}$.
18. $\frac{3}{34}$.
19. $\frac{32}{781}$.
20. $\frac{8}{3}$.
21. $\frac{187}{666}$.
22. $\frac{41}{14}$.
23. $\frac{48}{23}$.
24. $\frac{13}{138}$.
25. $\frac{5762}{1815}$.
26. $\frac{2071}{23742}$.
27. $\frac{1077}{709}$.
28. $\frac{81}{70}$.
29. $\frac{323}{8481}$.
30. $\frac{212}{1719}$.
31. $\frac{31}{471}$.
32. $\frac{545}{414}$.
33. $\frac{565}{2127}$.
34. $\frac{404}{573}$.
35. $\frac{329}{8786}$.
36. $\frac{212}{6167}$.
37. $\frac{7}{11}$.
38. $\frac{1}{25}$.
39. $\frac{47}{138}$.
40. $\frac{23}{323}$.
41. $\frac{4597}{21002}$.
42. $\frac{1}{264}$.

Point 102.

1. $5\frac{2918833}{6031780}$.
2. $2\frac{17}{36}$.
3. $1\frac{61}{280}$.
4. $2\frac{1277}{11440}$.
5. $2\frac{137}{281}$.
6. $2\frac{521}{900}$.
7. $8\frac{4427}{6160}$.
8. $14\frac{53267}{143115}$.
9. $2\frac{42617}{45045}$.
10. $2\frac{14548}{17765}$.
11. $\frac{2}{9}$.
12. $27\frac{3}{5}$.
13. $\frac{433}{630}$.
14. $3\frac{103}{1080}$.
15. $\frac{19}{100}$.
16. $13\frac{26}{45}$.
17. $4\frac{19}{20}$.

Point 106.

1. $\frac{10}{297}$.
2. $\frac{228}{1265}$.
3. $\frac{9}{55}$.
4. $\frac{648}{42625}$.
5. $\frac{1197}{2560}$.
6. $\frac{11}{362880}$.
7. $\frac{13}{342}$.
8. 0.

Point 110.

1. 25.
2. 8.
3. $3\triangledown 5$.
4. $29\triangledown 58\dot{3}$.
5. $16\triangledown \dot{3}68\,421\,052\,631\,578\,94\dot{7}$.
6. $35\triangledown \dot{3}$.
7. $37\triangledown 75$.
8. $73\triangledown \dot{1}$.
9. $6\triangledown 25$.
10. 7.
11. $6\,985\triangledown \dot{7}\dot{2}$.
12. $560\triangledown 91\dot{6}$.
13. $8\,474\triangledown \dot{1}$.
14. 3 130.
15. $517\triangledown \dot{1}42\,85\dot{7}$.
16. $12\triangledown 3\,\dot{5}71\,42\dot{8}$.
17. 286.
18. $452\triangledown \dot{6}92\,30\dot{7}$.
19. 239 091.
20. 2 000.

Point 114.

1. ▿4 7$\dot{7}\dot{2}$.
2. 4 128$\frac{4}{7}$.
3. 4$\frac{71}{84}$.
4. $\frac{3}{160}$.
5. $13▿578 125.
6. ▿4 91$\dot{6}$.
7. 506▿0 575 Dm.
8. 21 400.
9. 827▿034 583.
10. 14 939$\frac{63}{83}$.
11. $\frac{4}{915}$.
12. $\frac{1}{248}$.
13. $\frac{3}{40}$.
14. 305 016.
15. 133.
16. 57 024.
17. 1 144.
18. 22 304.
19. 21.
20. 3▿2.
21. 115$\frac{581}{999}$.
22. ▿96.
23. 4$\frac{107}{196}$.
24. 1$\frac{251}{405}$.
25. ▿8.
26. 5$\frac{13}{22}$.
27. 1$\frac{1}{49}$.
28. ▿4.
29. ▿5.
30. $\frac{259}{1440}$.
31. a. 200 dm;
    b. 225 dm.
32. 2$\frac{3623}{36000}$.
33. 3$\frac{81}{256}$.
34. 1$\frac{859}{2176}$.
35. 35$\frac{175}{216}$.
36. $\frac{97}{504}$.
37. 10$\frac{17}{160}$.
38. 206 000 034 444 024 003▿00000 000 000 017 000 282.
39. 139 448 900.
40. 1$\frac{1491183}{2577300}$.
41. 7 868 154$\frac{603}{608}$.
42. 107$\frac{61}{140}$.
43. $\frac{437}{16494}$.
44. $\frac{3025}{306131}$.
45. 2$\frac{1343571257}{3568172000}$.
46. 17$\frac{7}{16}$.
47. 6▿463 883 888.
48. ▿1$\dot{6}$.
49. a. $25▿89;
    b. $51▿78;
    c. $155▿34.
50. 142$\frac{1289}{3080}$.
51. 247$\frac{7}{17}$.
52. 3 175.
53. 6 930.
54. 4 158.
55. 78 934 016 821▿9 940 73$\dot{7}$ mg.
56. 17▿70 $\dot{5}$47 93$\dot{7}$.
57. 117.
58. 133▿4$\dot{1}\dot{9}$.
59. 27 T.
60. 110.
61. 9▿6.
62. 2^9, 5, and 3.

63. \$117▿15.
64. \$217▿287948.
65. \$324▿06.
66. 1.
67. $4\frac{439}{462}$.
68. \$181▿975.
69. $\frac{5}{66}$.

Point 116.

1. a. 4;
b. $\frac{1}{4}$.
2. a. 100 000;
b. $\frac{1}{100000}$.
3. a. $1\frac{1}{2}$;
b. $\frac{2}{3}$.
4. a. 10;
b. $\frac{1}{10}$.
5. a. 450;
b. $\frac{1}{450}$.
6. a. 1;
b. 1.
7. a. 576;
b. $\frac{1}{576}$.
8. a. $\frac{1}{48}$;
b. 48.
9. a. 40 000;
b. $\frac{1}{40000}$.
10. a. 1 000 000;
b. $\frac{1}{1000000}$.
11. a. 4;
b. $\frac{1}{4}$.
12. a. $\frac{7}{11}$;
b. $\frac{11}{7}$.
13. a. $\frac{1}{9}$;
b. 9.
14. a. 3;
b. $\frac{1}{3}$.
15. a. 3;
b. $\frac{1}{3}$.
16. a. $\frac{43}{5}$;
b. $\frac{5}{43}$.
17. a. 4;
b. $\frac{1}{4}$.
18. a. $\frac{1}{5}$;
b. 5.
19. a. 5;
b. $\frac{1}{5}$.
20. a. 7;
b. $\frac{1}{7}$.
21. a. 12;
b. $\frac{1}{12}$.
22. a. 17;
b. $\frac{1}{17}$.
23. a. $\frac{1}{4}$;
b. 4.
24. a. $\frac{23}{199}$;
b. $\frac{199}{23}$.
25. a. $\frac{199}{23}$;
b. $\frac{23}{199}$.
26. a. 7;
b. $\frac{1}{7}$.
27. a. 750;
b. $\frac{1}{750}$.
28. a. 3;
b. $\frac{1}{3}$.

Point 128.

1. 1 250.
2. \$▿$\dot{2}$38 09$\dot{5}$.
3. 937▿5 mi.
4. 2▿6 91$\dot{6}$.
5. a. 125 032;
b. 31 258.
6. 280.
7. \$145▿01$\dot{6}$.
8. $9\,339\frac{3}{7}$.
9. 1 050.
10. 7▿745 Ha.
11. \$10 958▿$\dot{9}$0 410 95$\dot{8}$.
12. \$18▿25.
13. 36▿1 625.
14. \$$6\,428\frac{4}{7}$.
15. \$44▿62 125.
16. 50▿7.
17. \$11▿$\dot{6}$.
18. $166\frac{2}{13}$ cm.
19. \$48.
20. \$$530\frac{10}{13}$.
21. \$46▿875.
22. 12 400.
23. \$30▿$\dot{2}\dot{7}$.
24. \$118▿44.
25. 64▿$\dot{6}\dot{3}$.
26. \$109▿3 125.
27. 156▿8 m.
28. 48▿6 d.
29. 872▿73.
30. 16▿5 d.
31. 3▿4$\dot{0}\dot{9}$.
32. \$52▿2.
33. 4 995 000.
34. 180 000.
35. 41 875.
36. ▿1 625.
37. 72.

38. 340.
39. 2 112 Cm.
40. 64 Cg.
41. $▿1̇94 805̇.
42. 5▿9̇25̇ Cg.
43. $42 000.
44. $80▿25.
45. $28.
46. ▿05̇.
47. $470▿0 913.
48. $9\frac{9}{14}$.
49. 49▿5 s.
50. $50▿3̇600̇.
51. $209▿0̇9̇.
52. $700.
53. $1▿33.
54. $157▿027.
55. $169▿325.
56. 19 130▿155 s.
57. $100.
58. $4 166▿7.
59. 191 st.
60. $400.
61. 893▿75.
62. 12 296▿34.
63. 12.
64. 506▿6̇.
65. 13▿75.
66. $43\frac{3}{4}$.
67. 154.
68. 1▿112̇.
69. $1\,714\frac{2}{7}$.
70. 22▿7̇2̇.
71. 10.
72. $189.
73. 439▿2.
74. 162.
75. 42.
76. 12.
77. $2.
78. $139▿65.
79. $195▿52.
80. 3 150▿.
81. 8 173▿2.
82. 793▿6̇50 793̇ d.
83. 74 446▿0̇2 739 726̇.
84. 480.
85. 90 000.
86. $72.
87. 2 916▿6̇.
88. $83▿3̇.
89. $1▿7 67̇2̇.
90. 46▿6̇.
91. 1 800.
92. 5 000.
93. $101\frac{13}{15}$.
94. 16.
95. 100.
96. $53▿55.
97. 67 h.
98. 1 363▿2.
99. 19 000.
100. $59\frac{3}{8}$.
101. $8 000.
102. ▿8 9̇54̇.
103. $12.
104. $60▿5̇.
105. 2▿82 mi.

106. $10.
107. 2▿5 y.
108. 1 282▿05.
109. 45▿6.
110. 5▿3̇.
111. 67▿5 Mm.
112. 17▿2̇7̇ d.
113. 24 000.
114. 2▿8̇.
115. 39▿6̇.
116. $304\frac{8}{23}$.
117. 700▿8.
118. 9.
119. 1 014▿1̇09̇.
120. 14▿0 625 Q.
121. $292▿6̇8 292̇.
122. 33 750.
123. $34\frac{7}{8}$.
124. 937▿8.
125. $325.
126. $5\frac{1}{4}$ d.
127. $2\frac{1}{25}$ y.
128. 10.
129. ▿1̇85̇.

Point 132.

1. 8.
2. 52 000.
3. 54▿5̇4̇.
4. $493\frac{7}{9}$.
5. $150\frac{6}{7}$.
6. ▿120 095̇.
7. 26▿25.
8. $17\frac{1}{2}$.
9. 120.
10. 1 617.
11. 343.
12. 57▿100 5̇55 661 872 493̇.
13. 9▿48 Hg.
14. 3▿725 m.
15. $584▿72.
16. $6\frac{1}{8}$.
17. a. The 2d;
b. the 4th;
c. the 2d and 3d;
d. the 1st and 4th;
e. $1\frac{557}{616}$.
18. $\frac{859}{2436}$.
19. $79.
20. 1▿96 416̇.
21. $\frac{76}{50875}$.
22. 52.
23. 20 000.
24. ▿06 399.
25. ▿122.
26. 797▿5 9̇04 761̇ m.
27. 188 100.
28. 220.
29. $414\frac{11917}{16650}$.
30. $45▿36.
31. a. $539;
b. $147;
c. $42.
32. $12\frac{5}{6}$.
33. $5\frac{8}{17}$.
34. $2\frac{3}{16}$.
35. 31▿948̇.
36. 20▿8.

37. 11 106▿312 m.
38. $\frac{2}{5}$.
39. $2\frac{253}{336}$.
40. 225.
41. $713\frac{13188}{70875}$.

Point 135.

1. $3 827▿25.
2. $1 747▿2.
3. $10 721▿33 723.
4. $217▿065.

Point 140.

1. $63 926▿568.
2. $28 481▿1.
3. $7 197▿9.
4. $13 640.
5. $23 110.
6. $4 416▿3.
7. $2▿017 501.
8. $15 837▿66.
9. $91▿275.
10. $23 475▿2.
11. $330▿01.
12. $737▿325.
13. $1 136▿2 395.
14. $225.
15. $23▿8 995.
16. $7 386▿22.
17. $1▿17.
18. $231▿68.
19. $21▿63.
20. $6 771▿75.
21. 70 190.
22. $3 685▿1 088.
23. 834▿143.
24. $6 885▿835.
25. 1.
26. 126▿217 Dl.
27. 239.
28. 5 040.
29. 79▿7 896̇.
30. $10▿023.
31. ▿4 584.
32. 3 663▿79 144 8̇19̇.

Point 143.

1. ▿113.
2. 1▿2 425.
3. ▿005.
4. ▿23 625.
5. ▿193̇.

Point 146.

1. $190▿71.
2. 3 768▿9 075 l.
3. $37▿105.
4. $110.
5. $73▿419.
6. 548.
7. a. $749;
   b. $1 273▿3;
   c. $2 022▿3;
   d. $3 445▿4.
8. $11▿775.
9. 648▿81 Dl.
10. $1 097▿985 625.

11. a. 20%;
  b. $7 494▿47;
  c. $6 448▿73;
  d. $3 485▿8.
12. a. 83%;
  b. 24 734;
  c. 5 066.
13. $37▿76 175.
14. $29▿5$\dot{6}$.
15. $152▿85 465.
16. $217▿57 195.
17. $844▿95 875.
18. $157▿8 774.
19. $70.
20. $742▿06 125.

Point 148.

1. a. $4 000;
  b. $80.
2. a. $311▿08;
  b. $7 777.
3. a. 11 523$\frac{17}{21}$;
  b. $8 066▿$\dot{6}$;
  c. 403▿$\dot{3}$.
4. a. $10 800;
  b. $94▿5.
5. a. $500;
  b. $12 500;
  c. 347▿$\dot{2}$.
6. $91▿25.
7. $150.
8. $7 328▿125.
9. $4 860.
10. $8▿9.
11. $1 111$\frac{1}{2}$.

Point 151.

1. $6 200.
2. $20 000.
3. $24 250.
4. $1 600.
5. $35 000.

Point 155.

1. $7 142$\frac{6}{7}$.
2. $18 532.
3. $11 111▿$\dot{1}$.
4. $7 837$\frac{31}{37}$.
5. $15 712.
6. $20 618$\frac{63}{97}$.
7. $19 999▿5.

Point 158.

1. $1 005▿8.
2. $167▿7.
3. $2 467▿3 488.
4. $223▿23.
5. $9▿24.
6. 113▿552.
7. a. $279;
  b. $112▿5;
  c. $171.

Point 162.

1. $354▿68.
2. $1 219▿35.
3. 5 y.
4. 3$\frac{7}{9}$%.
5. 4$\frac{38}{63}$ y.
6. $1 540▿2.
7. $605▿32.

8. $8\frac{1}{2}$%.
9. $17\frac{109}{1998}$ y.
10. $4\frac{172}{2457}$.
11. $450▿32.
12. 5%.
13. $165▿8.
14. 42 c.
15. $404▿52.
16. $63▿55.
17. $27 802▿31.
18. $15 932▿32.
19. $183▿17.
20. $11▿48.
21. $3 366▿61.
22. $29 324▿71.
23. $21▿44.
24. $274▿18.
25. $306▿92.
26. $927▿55.
27. $1▿04.
28. $410▿54.
29. $4 113▿5.
30. $69▿74.
31. $1 508▿49.
32. 6%.
33. $27▿7.

Point 165.

1. $95▿07.
2. $338▿35.
3. $891▿13.

Point 170.

1. $950▿12.
2. $1 841▿58.
3. $1 001▿4.
4. $441▿45.
5. $49 211▿43.
6. $3 968▿81.
7. $1 501▿91.
8. $1 230▿1.
9. $159▿92.

Point 174.

1. In 9 mo.
2. $3\frac{7}{18}$ mo.
3. In $4\frac{1}{2}$ mo.
4. $4\frac{1}{2}$ mo.
5. In 4▿36 mo.
6. September 22d.

Point 177.

1. 78 125.
2. 2 744.
3. 64.
4. $625\frac{31149}{32768}$.
5. $\frac{9}{25}$.

Point 181.

1. 17▿489 333 100 625.
2. $317\frac{5299}{6561}$.
3. 4^{25}.
4. 12.
5. 18^{4}.
6. 6^{5}.
7. 7^{49}.
8. $(\frac{7}{2})^{54}$.
9. 8^{7}.
10. $\frac{24389}{1000}$.
11. 1 764.

Point 184.

1. 3^5.
2. 9^2.
3. 5.
4. 11^6.
5. 21^5.
6. $\frac{1}{216}$.

Point 196.

1. 72.
2. 5▿2 Hm.
3. $67▿2.
4. 4▿5 T.
5. ▿45.
6. 60▿8.
7. 2▿1 m.
8. 274.
9. 35.
10. 2.
11. 26.
12. 13.
13. 4▿ dm.
14. 1▿8 dm.
15. $25▿5.
16. $119▿5.
17. 1▿130 973 354+ ca.
18. 523▿598 775+ Cs.
19. a. 138 095 716▿823 595+ ms.
    b. 100 Ms.
20. $86▿4.
21. 442.
22. ▿26.
23. 992.
24. 2▿2 360 678.
25. 7▿7 773 471+.
26. ▿00 111.
27. 282 475 249.
28. ▿$\dot{6}$.
29. 2▿267 786+.
30. 3▿$\dot{6}$.
31. 66.
32. 432.
33. 85.
34. 242.
35. 98.
36. 115.
37. 109.
38. 997.
39. 5 464.
40. 1 432.
41. ▿953 462+.
42. ▿866 025+.
43. $\frac{25}{81}$.
44. $\frac{24}{135}$.
45. ▿035.
46. 14▿01.
47. 1▿50 051.
48. 7▿625.
49. 4▿213 074+.
50. 3▿00 001.
51. 59 049.
52. 103▿4.
53. 73.
54. 2▿7 Dm.
55. 10▿08 Cm.
56. 19 Hm.
57. 158 345 cm.

58. 145▿5 m.
59. 6▿633 249 + m.
60. 5 015▿974 481 + mm.
61. 19 078▿784 028 + mm.
62. 10▿121 264 + Mm.
63. 141▿421 356 + mm.
64. 31▿176 914 + m.
65. 41▿301 537 + m.
66. 420.
67. 376.
68. 14▿142 135 + m.
69. 2 m.
70. 1▿732 068 + m.
71. 5▿641 895 + Dm.
72. 69.
73. 3 973.
74. 125.
75. 1 025.
76. ▿887.
77. 46▿8.
78. 56▿42.
79. 7▿85.
80. ▿86.
81. $\frac{2}{5}$.
82. ▿560 906 +.
83. 3▿063 462 +.
84. 3▿2.
85. 42▿7.
86. 74.
87. 234.
88. 25.
89. 55.
90. 101.
91. 165.
92. 344▿3.
93. $\frac{9}{11}$.
94. 2▿34.
95. ▿0 809.
96. $\frac{1}{5}$.
97. ▿557.
98. 8 m.
99. 36 cm.
100. 12 150.
101. 5▿5 m.
102. 4 Dm.
103. 3▿7 $\dot{9}2\dot{5}$ Q.
104. $51 450.
105. 610▿214 250 7 + Cg.
106. 9▿173 463 + Hm.
107. 32.
108. 1▿1.
109. 13.
110. 3.
111. 376.
112. 2 197 s.
113. 1 Mm.
114. 2▿91 Cm.
115. 1 237▿0 021 059 375 + ca.

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